M E T U
Northern Cyprus Campus

	Differential Equations **I. Midterm**	
Code : *Math 219* Acad.Year: *2007-2008* Semester : *Fall* Date : *4.11.2007* Time : *14:00* Duration : *120 minutes*	Last Name: Name : Department: Signature: Student No.: Section:	
	5 QUESTIONS ON 5 PAGES TOTAL 100 POINTS	

1	2	3	4	5		

1. (10+10=20 points) Solve

(a) $y' = \dfrac{x - \sin x}{y^4 + 2}$.

(b) $(3yx^2)dx + (x^3 + 2y^4)dy = 0$, $y(0) = 2$.

2. (7+10+3=20 points) A tank initially contains 60ℓ of pure water. A solution containing $1g$ of salt per liter enters the tank at $2\ell/min$, and the perfectly mixed solution leaves the tank at $3\ell/min$, therefore the tank is empty after 1 hour.

(a) Express the volume of solution in the tank in terms of t, and write a differential equation for the amount of salt in the tank at time t (Hint: the resulting equation will be a first order linear equation, but not a separable equation).

(b) Find the amount of salt in the tank after t minutes.

(c) What is the maximum amount of salt ever in the tank?

3. (10+10=20 points) Solve

(a) $y'' + 3y' - 4y = 2x - e^x$.

(b) $y'' + y = 4 \sin x$.

4. (20 points) If the functions y_1 and y_2 are linearly independent solutions of $y''+p(t)y'+q(t)y = 0$, prove that $y_3 = y_1 + y_2$ and $y_4 = y_1 - y_2$ also form a linearly independent set of solutions.

5. (4+16=20 points) (a) Check that $y = x$ is a solution of $x^2y'' - 2xy' + 2y = 0$

(b) Find all solutions of $x^2y'' - 2xy' + 2y = x^3 \ln x$.