	Differential Equations **Final**	
Code : *Math 219* Acad.Year: *2007-2008* Semester : *Fall* Date : *7.1.2008* Time : *16:00* Duration : *120 minutes*	Last Name: Name : Department: Signature:	Student No.: Section:
	5 QUESTIONS ON 5 PAGES TOTAL 100 POINTS	

1	2	3	4	5		

1. (20 pts.)Let

$$\mathbf{x}' = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} t^{-1} \\ 2t^{-1} + 4 \end{bmatrix}, \ t > 0$$

where $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Find the general solution of this system of equations.

2. (6+7+7=20 points) Solve the initial value problem $y'' + 4y = 0\,,\ \ y(0) = 0\,,\ \ y'(0) = 0$ in the following <u>three</u> different ways:

(a) Using the characteristic equation method

(b) Using the Laplace Transform method

(c) Using a power series expansion around $x_0 = 0$. Show that your answers in *a*. *b*. and *c*. are equal.

3. (10+5+5=20 points) Suppose that $f(t) = t$, for $0 < t < 1$, and f is odd of period 2.

(a) Show that $f(t) = 1 - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin(n\pi t)}{n}$ at the points that f is continuous.

(b) Sketch the graph of the Fourier series of f indicating the value at each discontinuity.

(c) Use (a) to show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$ $\left(\text{Hint: let } t = \frac{1}{2}\right)$

4. (10+10=20 points) Solve the system of linear differential equations;

(a)

$$\frac{dx}{dt} = x - 5y$$
$$\frac{dy}{dt} = 2x - y$$

(b)

$$\frac{dx}{dt} = x + 2y$$
$$\frac{dy}{dt} = 4x + 3y$$

5. (10+10=20 points) (a) Solve the following differential equation:

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

(b) Find the values of λ such that

$$y'' + 2y' + \lambda y = 0\,, \quad y\,(0) = 0\,,\ y\,(1) = 0$$

has a non-trivial solution.