IMPROVING THE NUMERICAL EFFICIENCY OF THE METHOD OF MOMENTS FOR PRINTED GEOMETRIES

Lale Alatan*[1], M. I. Aksun[2] and Tuncay Birand[1]

1 Middle East Technical University,Dept. of Electrical & Electronics Eng. 06531 Ankara Turkey

2 Bilkent University,Dept. of Electrical & Electronics Eng. 06533 Ankara Turkey

1 Introduction

The rigorous and numerically efficient analysis of printed geometries in a multilayer medium has been a major research topic in the computational electromagnetics because of the common use of these structures in the monolithic microwave integrated circuits and in the design of printed antennas. Some numerical techniques like Finite Difference Time Domain (FDTD), Finite Elements Method (FEM), and Method of Moments (MoM) in the spatial and spectral domains, are commonly used in the analysis of such planar geometries. Although the FDTD and FEM are numerically rigorous and versatile, they are computationally very expensive. Recently, it has been demonstrated that the use of the spatial domain MoM in conjunction with the recently developed closed-form Green's functions for the solution of the mixed-potential integral equation significantly improves the computationally efficiency[1]-[3]. This improvement is due to the elimination of the numerical evaluation of the slow-convergent and highly oscillatory Sommerfeld integral by approximating it with a finite series of complex functions. This approach results in two-dimensional integrals over finite domains with the smooth integrands as the MoM matrix elements. In this paper, a technique of reducing the remaining double integrals to single integrals is presented and the improvement in the computational efficiency is demonstrated for a microstrip line.

2 Formulation

The formulation presented herein is applicable to general microstrip geometries in a multilayer medium where it is assumed that the layers extend to infinity in the transverse directions. However, for the sake of illustration, the formulation is presented for a microstrip line over a substrate, for which only the longitudinal current is assumed to exist.

The mixed-potential integral equation for the microstrip line can be transformed into the matrix equation with the use of the well-known MoM proce-

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dure, and a typical matrix element is given below to help demonstrate the use of the formulation;

$$< T_{xm}, G^A_{xx} * B_{xn} > + \frac{1}{\omega^2} < T_{xm}, \frac{\partial}{\partial x}\left(G_q * \frac{\partial}{\partial x} B_{xn}\right) > \qquad (1)$$

where T_{xm}, B_{xn} are the testing and basis functions, respectively, and G^A_{xx}, G_q are the Green's functions of the vector and scalar potentials, respectively. The first inner product of (1) is written explicitly as

$$\begin{aligned} &< T_{xm}, G^A_{xx} * B_{xn} >= \\ &\qquad \int_{D_T}\!\int dx dy T_{xm}(x,y) \int_{D_B}\!\int dx' dy' G^A_{xx}(x-x', y-y') B_{xn}(x', y') \end{aligned} \qquad (2)$$

where D_T and D_B denote the domains of the testing and basis functions, respectively, and the closed-form Green's function G^A_{xx} is expressed as

$$G^A_{xx}(x-x', y-y', z=0) = \sum_{i=1}^{N} c_i \frac{e^{-jk\sqrt{(x-x')^2+(y-y')^2+A_i^2}}}{\sqrt{(x-x')^2+(y-y')^2+A_i^2}} \qquad (3)$$

with the complex constants c_i and A_i obtained from the generalized pencil of function method which is used to approximate the integrand of the Sommerfeld integral in terms of complex exponentials. By changing the order of integration, the inner product takes the form of

$$\int\int du\, dv\;\; G^A_{xx}(u,v) \underbrace{\int\int dx\, dy\; T_{xm}(x,y)\; B_{xn}(x-u, y-v)}_{\text{correlation function } (T_{xm} \otimes B_{xn})}. \qquad (4)$$

The basis and testing functions are chosen to be the rooftop functions which are triangular functions in the longitudinal direction and uniform in the transverse direction. For this choice, the correlation function becomes

$$T_{xm} \otimes B_{xn} = f(u) g(v) \qquad (5)$$

where $f(u) = (\alpha_3 u^3 + \alpha_2 u^2 + \alpha_1 u + \alpha_0)$ for $\beta_1 h_x < u < \beta_2 h_x$

$$g(v) = \frac{1}{W^2}\begin{cases} v + W & -W < v < 0 \\ -v + W & 0 < v < W \end{cases}$$

and $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2$ are constants determined by m, n, h_x (the half-length of the current cell.) Since $g(v)$ is a polynomial of degree one, the integral with respect to v is chosen to be performed analytically. By substituting the correlation function (5) and the Green's function (3) into (4), the inner-product term can be written as

$$\sum_{i=1}^{N} c_i \int du f(u) \left[\underbrace{-\frac{2}{W^2}\int_0^W \frac{e^{-jkR}}{R} v dv}_{I_1} + \underbrace{\frac{1}{W^2}\int_{-W}^{W} \frac{e^{-jkR}}{R} dv}_{I_2} \right] \qquad (6)$$

where $R = \sqrt{u^2 + v^2 + A_i^2}$. The first integral can be integrated analytically by using the substitution method, and is given as

$$I_1 = \frac{2j}{W^2 k}\left(e^{-jk\sqrt{u^2+A_i^2}} - e^{-jk\sqrt{u^2+W^2+A_i^2}}\right). \tag{7}$$

However, the evaluation of the second integral is not straightforward, which requires e^{-jkR} to be approximated by its Taylor series expansion in two different regions[4]. The first region includes the points close to the source, whereas the second region is chosen to be the Fraunhofer region starting at the Rayleigh distance, $RD = 2W^2/\lambda$. The term e^{-jkR} is expanded around $R = 0$ in the first region and around $R = r$ in the second region where $r = \sqrt{u^2 + A_i^2}$, and the number of terms in each region is determined by setting an error criterion. Consequently, the integral I_2 is obtained analytically as

Region1: $0 < r < RD$

$$\int \frac{e^{-jk\sqrt{r^2+v^2}}}{\sqrt{r^2+v^2}}dv = \ln\left|v + \sqrt{r^2+v^2}\right| - jkv, \tag{8}$$

Region2: $r > RD$

$$\begin{aligned}\int \frac{e^{-jk\sqrt{r^2+v^2}}}{\sqrt{r^2+v^2}}dv = e^{-jkr}\Bigg[&\ln\left|v + \sqrt{r^2+v^2}\right|\left(1 + jkr - \frac{k^2r^2}{2} - j\frac{k^3r^3}{6}\right)\\ &+v\left(-jk + k^2r + j\frac{k^3r^2}{2}\right) + j\frac{k^3}{6}\left(r^2v + \frac{v^3}{3}\right)\\ &+\frac{1}{2}\left(v\sqrt{r^2+v^2} + r^2\ln\left|v+\sqrt{r^2+v^2}\right|\right)\left(-\frac{k^2}{2} - j\frac{k^3r}{2}\right)\Bigg].\end{aligned} \tag{9}$$

The same procedure can be applied to the second inner-product term of (1) in which G_q has the same functional form as G_{xx}^A given in (3). Note that the correlation function must be polynomial in form in order for the above formulation be applicable.

3 Results and Conclusion

In this part of the study, the formulation described above is applied to a microstrip line and the improvement in the CPU time, compared to the analysis with the double integrals, is observed. The dielectric constant of the medium is $\epsilon_r = 4.0$, the width of the line W to the thickness of the substrate d ratio is 4.0, the thickness of the substrate is 0.02032 cm (=8.0 mils.), the frequency is 1 GHz, the length of the line is 10cm, and the source is located 2 cm from left. The CPU times are obtained for SUN Sparc 2 system.

The current distribution on the microstrip line is obtained, first by numerically integrating the double integrals (Case 1), Eq. (6), involved in the MoM matrix elements, and second by using the equations given above (Case 2), Eqs.

(7)-(9). The Gauss quadrature integration algorithm is employed for both the double and single integrals with the same number of function evaluation for the u-integrations, for the sake of fairness. In addition, the v-integration in the first approach is performed with the least possible function evaluations, resulting the current distribution with an acceptable error. It is observed from Table 1 that the proposed method saves two-third of the computation time for large number of basis functions. Next, the number of evaluation points for the single integral case is reduced to one-half (Case 3), due to the fact that the outer intagrand varies smoother after having integrated the inner integrand analytically. Hence, the computation time is reduced approximately to the ten percent of the double integration case without sacrificing the accuracy, Fig. 1. As a result, it is observed that the formulation proposed here saves significant amount of computation time.

# of basis func.	Case 1	Case 2	Case 3
10	100.2	39.3	20.4
20	200.3	67.7	28
30	289.9	93.8	34.2
40	395.2	121.4	40.8

Table 1: CPU times for different number of basis functions.

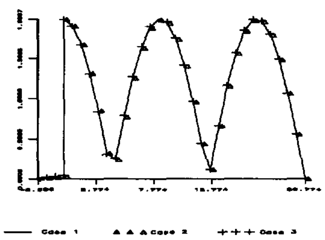

Figure 1: The current distribution

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