EE 503
Homework #3
(Due : Dec. 25, 2012)

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We examine the channel equalization problem in this assignment. Related discussions can be found in digital communications textbooks and also in [pages 530-534, Hayes].

Problem:

In the following set-up $h_{ch}[n]$ shows the impulse response of a causal FIR channel. We would like to design $h_{eq}[n]$, which is the equalizer filter, to compensate the effect of the channel. The process $w[n]$ is zero mean, white *Gaussian* noise (WGN) process with the variance σ_w^2. The process $s[n]$ is the information carrying process that we are interested in. The processes $s[n]$ is uncorrelated with $w[n]$.

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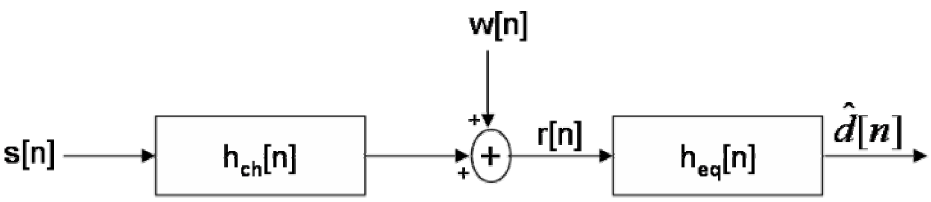

Figure: Scheme for channel equalization

a) Consider $H_{ch}(z) = h_{ch}[0] + h_{ch}[1]z^{-1} + \ldots + h_{ch}[L]z^{-L}$ and $d[n] = s[n-\Delta]$ where Δ is a non-negative integer. Analytically find the auto-correlation sequence $r_r[k]$ and the cross-correlation sequence $r_{dr}[k]$.

b) Take $H_{ch}(z) = \frac{14}{5} + \frac{2}{5}z^{-1} + \frac{4}{5}z^{-2} + 2z^{-3} + \frac{1}{5}z^{-4} - \frac{1}{5}z^{-5}$ and design FIR equalizers of order P, i.e. $H_{eq}(z) = h_{eq}[0] + h_{eq}[1]z^{-1} + \ldots + h_{eq}[P]z^{-P}$, such that $E\left\{\left(d[n] - \hat{d}[n]\right)^2\right\}$ is minimized.

Note that if it is possible to reduce the error of $E\left\{\left(d[n] - \hat{d}[n]\right)^2\right\}$ to zero (ideal equalizer), then the cascade of $h_{ch}[n]$ and $h_{eq}[n]$ becomes $\delta[n-\Delta]$; hence after the equalization the cascade channel becomes the delay channel with colored additive noise.

For the design of the filters assume that $r_s[k] = \delta[k]$ and $r_w[k] = \frac{1}{SNR}\delta[k] = \sigma_w^2\delta[k]$

Here $SNR = \frac{r_s[0]}{r_w[0]} = \frac{\sigma_s^2}{\sigma_w^2}$ shows the ratio of signal and noise powers.

Submit following :

1. Plot of $conv(h_{ch}[n], h_{eq}[n])$ at SNR = 10 dB, $\Delta = 2$ for $P = \{3,5,7,11\}$. (Use subplot command of Matlab, if necessary).
2. Plot of $conv(h_{ch}[n], h_{eq}[n])$ at SNR = 10 dB, $\Delta = 4$ for $P = \{3,5,7,11\}$.

3. Using Matlab calculate the error of $J_{\min} = E\left\{\left(d[n] - \hat{d}[n]\right)^2\right\}$ of the optimum equalizer and fill in the following table for each pair of P and Δ at SNR = 10 dB.

Jmin	Δ=3	4	7	8	11	Δ=12
P=3						
4						
7						
8						
10						
11						
P=12						

Achieved Minimum Error for Various Configurations for SNR = 10 dB

4. Calculate the error of $J_{\min} = E\left\{\left(d[n] - \hat{d}[n]\right)^2\right\}$ of the optimum equalizer and fill in the following table for each pair of P and Δ at SNR = 0 dB.

Jmin	Δ=3	4	7	8	11	Δ=12
P=3						
4						
7						
8						
10						
11						
P=12						

Achieved Minimum Error for Various Configurations for SNR = 0 dB

Questions:

- For the submission of parts 3 and 4 comment on the effect of SNR. Is the system significantly affected by SNR?
- State the values for P and Δ for which the equalizer is all zero filter. These cases should appear in the tables with J_{min}=1. Explain whether this makes sense or not.

Note:

c) In this part, we numerically examine the success the effect of equalization on a communication scheme. This part shows how the equalizer is used in a practical set-up.

For this part, assume that $s[n]$ is equally likely to be 1 or -1 for every n and $s[n]$ is i.i.d. distributed. Convince yourself that this leads to $r_s[k] = \delta[k]$.

1. Generate $s[n]$ for $n = \{0,1,...,99\}$ and implement the filtering scheme shown in the figure with $H_{ch}(z) = \frac{14}{5} + \frac{2}{5}z^{-1} + \frac{4}{5}z^{-2} + 2z^{-3} + \frac{1}{5}z^{-4} - \frac{1}{5}z^{-5}$, for $\Delta = 2$, $P = 5$ and SNR = 10 dB. Present $s[n-\Delta]$ and $\hat{d}[n]$ using subplot command of Matlab. Compare $s[n-\Delta]$ and $\hat{d}[n]$. Do you think channel equalization is successful?

To get a numerical value for the success of equalization, empirically estimate the probability of error by counting how many times $\mathrm{sign}(\hat{d}[n]) \neq s[n-\Delta]$. What is your estimate for the probability of the error (probability of an erroneous case)?

Note: Since $s[n-\Delta] = \pm 1$, please use the following modified form of the sign function.

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$$sign(x) = \begin{cases} 1 & x > 0 \\ \pm 1 \left(\text{with prob. } \frac{1}{2} \right) & x = 0 \\ -1 & x < 0 \end{cases}$$

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2. Repeat the earlier part for $s[n]$ for $n = \{0,1,...,99999\}$ and provide your empirical estimate for the probability of error (as in earlier part) for each pair of P and Δ at SNR values of 10 and 0 dB.

P{Error}	Δ=3	4	7	8	11	Δ=12
P=3						
4						
7						
8						
10						
11						
P=12						

Probability of Error Estimate for SNR = 10 dB

P{Error}	Δ=3	4	7	8	11	Δ=12
P=3						
4						
7						
8						
10						
11						
P=12						

Probability of Error Estimate for SNR = 0 dB