

EE 503  
Hw # 5  
(Fall 2006)

**Problems 1,2,3:** (30 pts) Problems 4.3, 4.6, 4.12 from Hayes.

**Problem 4:** (40 pts) In this exercise, we generate an AR processes realization and try to estimate the PSD of the process from the realization using two different methods. The methods used for the PSD estimation are called periodgram method and AR signal modeling approach.

AR process used in this experiment is the output of the filter with the following transfer function:

$$H(z) = \frac{b(0)}{1 + \sum_{k=1}^4 a(k)z^{-k}}$$

The input  $x(n)$  is zero mean, unit variance Gaussian distributed white noise process. The output sequence is labeled as  $y[n]$ . The filter coefficients are  $b(0) = 1$ ,  $a(1) = 0.7348$ ,  $a(2) = 1.8820$ ,  $a(3) = 0.7057$ ,  $a(4) = 0.8851$ .

i) Plot the true PSD of the process in decibel scale. (Use semilogy instead of plot command of Matlab). Compare your plot with the plot given in Example 4.7.4 on Page 199 of Hayes.

ii) Find the impulse response of the filter in Matlab. Using the impulse response compute output auto-correlation  $r_y(k)$ . What is variance of  $y[n]$  and first 4 lags of  $r_y(k)$ ,  $k = 1 \dots 4$  ?

iii) Generate a vector of length  $N = 32$  whose elements are independent and identically distributed zero mean and unit variance Gaussian random variables. (Use randn command) Use the random vector as input  $x[n]$  and generate a realization for  $y[n]$  process.

Form the convolution matrix for  $y[n]$  as in Eq 6.73 (page 308) of Therrien to estimate 5x5 auto-correlation matrix of  $y[n]$ . Compute the auto-correlation matrix estimate using the auto-correlation method. Compare your estimation for auto-correlation lags with the ones produced by the Matlab. (Using xcorr.m with "biased" option)

Take 1024 point FFT of the estimated auto-correlation sequence to estimate PSD. Plot the estimated PSD on the plot of true PSD, generated in part i).(Be careful in FFT operation, FFT assumes the input is in the range  $[0, N-1]$ ; while xcorr.m produces an output  $[-lags, lags]$ . You can not directly take FFT of xcorr output).

iv) For the  $y[n]$  realization generated in part iii), fit an AR model using auto-correlation method. Set AR model order to 6 poles. You can use "acm.m" given in Hayes' book to find the model parameters. Plot PSD of the estimated  $\hat{H}(z)$  (as done in part i). Compare the estimation with true PSD and the estimation generated in part iii.

v) Repeat parts iii) and iv) for  $N=64$  and  $N=128$ .

Submit only: Matlab code and plots for true PSD and two estimates for all  $N$ .