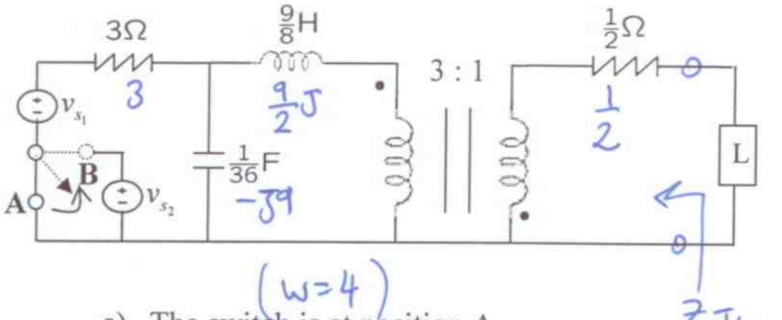
Question 1 (18 pts)



$v_{s_1}(t) = A\cos(4t)$ V,

$v_{s_2}(t) = A\cos(4t + 60^o)$ V

$(\omega=4)$

a) The switch is at position A.
The load is adjusted for the maximum power transfer.
The real power delivered to the load is 300 Watts.
Find the reactive power delivered to the load.

b) The switch is moved to position B.
Find the real power delivered to the load.

a) $Z_L = Z_{Th}^*$; $Z_{Th} = \left[(3 \| -j9) + \frac{9}{2}j\right]\frac{1}{9} + \frac{1}{2}$

$= \left[\left(\frac{1}{3} \| -j\right) + \frac{j}{2}\right] + \frac{1}{2}$

$= \left[\frac{-j/3}{\frac{1}{3} - j} + \frac{j}{2}\right] + \frac{1}{2}$

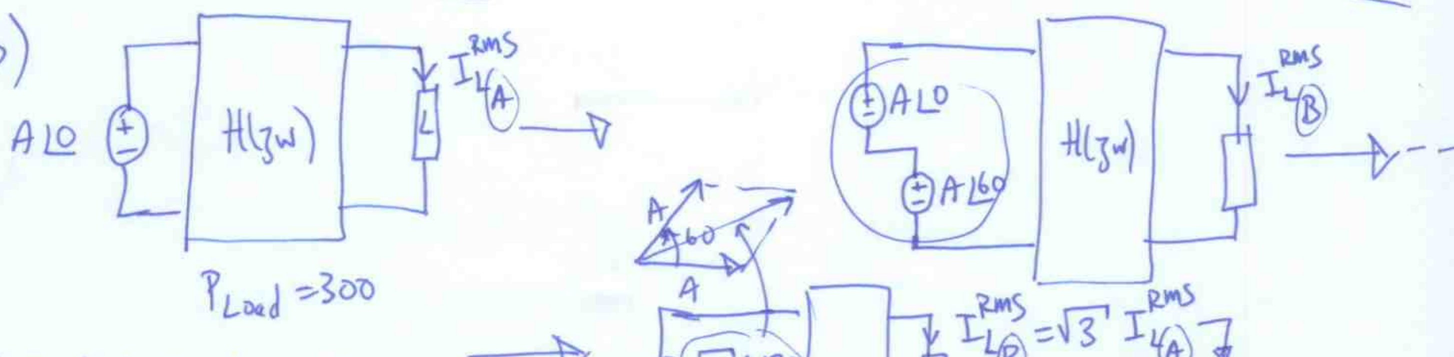
$= \left[\frac{-j(1+3j)}{10} + \frac{j}{2}\right] + \frac{1}{2}$

$= 0.8 + j0.4$

$Z_L = 0.8 - j0.4 \longrightarrow P_{Load} = (I_{Load}^{RMS})^2 \cdot (0.8) = 300$ Watt (given)

then $Q_{Load} = (I_{Load}^{RMS})^2 \cdot (-0.4) = -150$ Var.

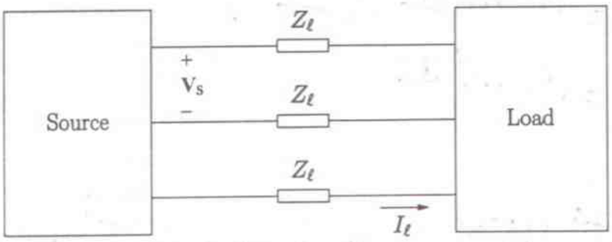
b)



$I_{L(B)}^{RMS} = \sqrt{3}\, I_{L(A)}^{RMS}$

$\longrightarrow P_{Load} = 3 \times 300 = 900$

Question 2 (24 pts) Consider the following balanced three-phase circuit with Δ-connected inductive load.



$P_{Load} = 14.4\text{kW}$,
$Z_\ell = \frac{1}{3} + j\frac{4}{9}\,\Omega$,
The percent efficiency, $\eta = 90\%$,
$V_{S,eff} = \frac{2000}{3\sqrt{3}}\ V_{rms}$.

Find:

+6 a) the effective value of the line current I_ℓ,

+6 b) the total complex power supplied by the source,

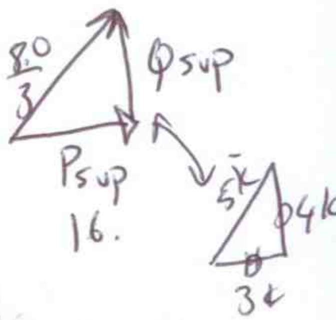
+6 c) the effective value of the line-to-line voltage at the load side,

+6 d) the per phase impedance of the load.

a) $\frac{P_{Load}}{P_{sup}} = 0.9 \rightarrow P_{sup} = \frac{14.4\text{kW}}{0.9} = 16\text{kW} \rightarrow P_{Line\,Loss} = 1.6\text{kW}$

$\rightarrow P_{Line_{Loss}} = 1600 = 3\left(I_\ell^{RMS}\right)^2 \frac{1}{3} \rightarrow \boxed{I_L^{RMS} = 40\text{ A RMS}}$ (a).

b) $|S_{sup}| = \sqrt{3}\, V_s^{RMS} I_L^{RMS} = \sqrt{3}\,\frac{2000}{3\sqrt{3}} \cdot 40 = \frac{80}{3}\text{kVA}$



$Q_{sup} = \frac{64}{3}\text{kVAR}$. | $\boxed{S_{sup} = 16 + j\frac{64}{3}\text{kVA}}$ (b)

c) $S_{Load} = S_{sup} - S_{Line} = \left(16 + j\frac{64}{3}\right) - \left(1.6 + j(1.6)\frac{4}{3}\right)$

$= 0.9\left(16 + j\frac{64}{3}\right)\text{kVA}.$

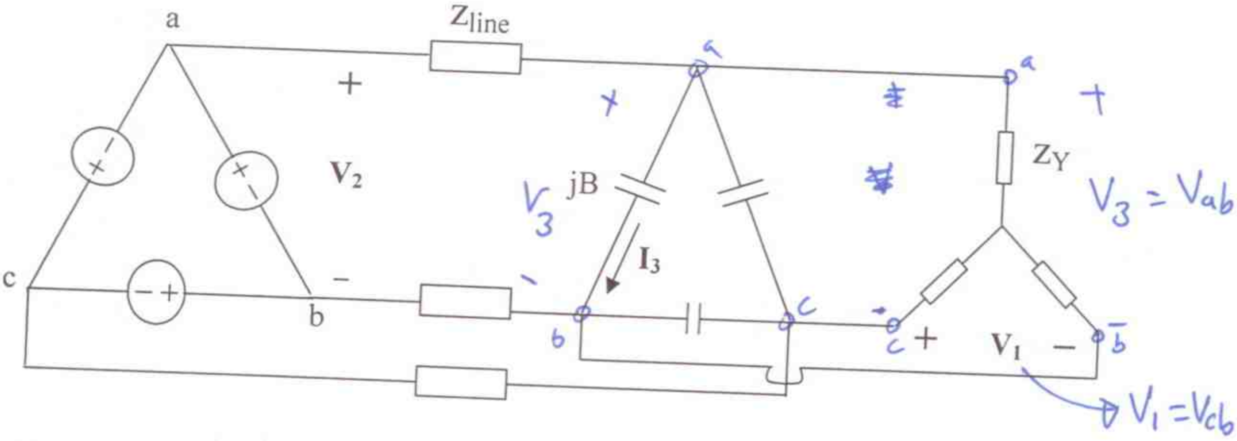
$|S_{Load}| = \sqrt{3}\left(I_L^{Rms}\right)\left(V_{Line,RMS}^{Load}\right) \rightarrow (0.9)\frac{80{,}000}{3} = \sqrt{3} \cdot 40 \cdot V_{Line\,RMS}^{Load}$

$\boxed{V_{Line}^{Load} = \frac{(2000)0.9}{3\sqrt{3}} = \frac{600}{\sqrt{3}} = 200\sqrt{3}\text{ V}}$

d) $S_{Load,\phi} = 0.3\left(16 + j\frac{64}{3}\right)10^3 = 40^2 Z_Y$

$\rightarrow Z_Y = 3 + j4 \rightarrow \boxed{Z_\Delta = 3Z_Y = 9 + j12\,\Omega}$

Question 3 (24 pts) Given a balanced 3-phase circuit with a positive phase sequence.



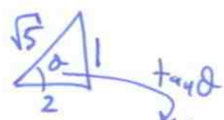
f = 50 Hz, $\quad Z_{line} = 0.4 + j1.2\ \Omega$, $\quad Y_Y = \frac{2}{9} - j\frac{1}{6}$ mhos

$V_1 = 180\sqrt{15}\angle 120°\ V_{rms}$.

The power factor of the Y load – capacitor bank combination is $\frac{2}{\sqrt{5}}$ lagging.

a) (12 pts) Find
   i) the complex power delivered to the Y-load,
   ii) the per phase capacitance of the capacitor bank,
   iii) the complex power supplied by the source,
   iv) the percent efficiency.

b) (12 pts) Find $i_3(t)$ and $v_2(t)$.



a) i) $S_{Load} = 3\frac{V_{p\phi}^2}{Z_Y^*} = 3\left(\frac{180\sqrt{15}}{\sqrt{3}}\right)^2\left(\frac{2}{9}+j\frac{1}{6}\right)$

$= 180.15\,(40+j30)$

$= 27{,}000\,(4+j3)$

ii) $S_{combi} = 27{,}000\,(4 + j4\cdot\tfrac{1}{2})$

$S_{cap} = -j27000$

$S_{cap\phi} = -j9000 = \frac{|V_1|^2}{Z^*_{cap}}$

$Z_{cap} = \frac{18\cancel{0}^2\cdot 15}{90\cancel{00}}(-j) = -j54$

$C = \frac{1}{2\pi 50\cdot 54} \cong \frac{1}{3.(5400)} = 60\mu F$

iii) $S_{combi} = \sqrt{3}\, V^{combi}_{Line}\, I_{Line}$

$|(27{,}000)(4+j2)| = \sqrt{3}\ 180\sqrt{15}\cdot I_{Line}$.

$\frac{(2\cancel{7{,}000})\cancel{4}^2}{2/\cancel{\sqrt{5}}}\cdot\frac{1}{\cancel{\sqrt{3}}}\cdot\frac{1}{\cancel{180}\cancel{\sqrt{15}}} = I_{Line}$.

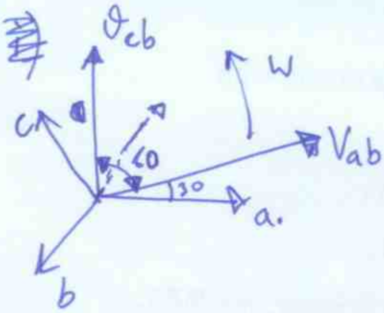
$I_{Line} = 100\ A\ (RMS)$

$S_{Line} = 3\, I_{Line}^2\,(0.4+j1.2) = 12 + j36$ kVA

$S_{sup} = (27\times 4 + j27\times 2)$
$\quad + (12 + j36)$ kVA.
$= 120 + j90$ kVA

iv) $\eta = \frac{(27)(4)}{120} = 90\%$

b)

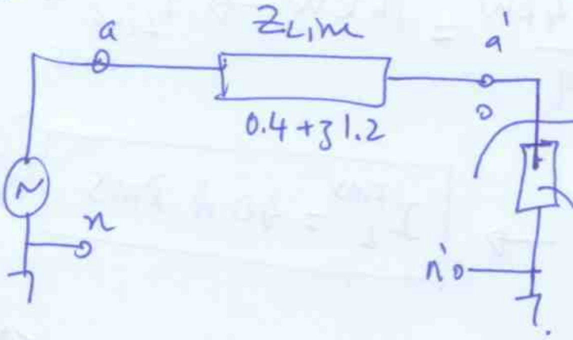


$$V_{ab} = V_{cb}\angle{-60^\circ}$$

$$V_{ab} = \left(180\sqrt{15}\angle{120}\right)\left(\angle{-60^\circ}\right)$$
$$= 180\sqrt{15}\angle{60}$$

$$i_3 = \frac{V_3}{-jB} = \frac{180\sqrt{15}\angle{60^\circ}}{-j54} = \frac{10\sqrt{5}}{\sqrt{3}}\angle{150^\circ} \longrightarrow i_3(t) = 10\sqrt{\frac{5}{3}}\cos(2\pi 50t + 150^\circ)$$ A_{RMS}

$V_2(t)$



$$V_{a'n'} = V_{a'b'}\angle{-30}$$
$$= 180\sqrt{15}\angle{30}$$

$$Z_{Tot} = Z_Y \,\|\, \left(\frac{-jB}{3}\right) \quad \leftarrow 54$$
$$= \left[\left(\frac{2}{9} - j\frac{1}{6}\right) + \left(\frac{+j}{18}\right)\right]^{-1}$$
$$= \left(\frac{2}{9}(1-j)\right)^{-1} = \frac{9}{2}\frac{(1+j)}{2}$$

$$\vartheta_{an} = \left(\frac{180\sqrt{15}\angle{30}}{\frac{9}{4}\sqrt{2}\angle{45}}\right)\left(\frac{9}{4} + j\frac{9}{4} + 0.4 + j1.2\right)$$

$$V_{an} = 80\sqrt{\frac{15}{2}}\angle{-15^\circ}\left(2.65 + j3.45\right)$$

$$\vartheta_2 = \vartheta_{an}\sqrt{3}\angle{+30^\circ} = 240\sqrt{\frac{5}{2}}\angle{15^\circ}\left(2.65 + j3.45\right)$$

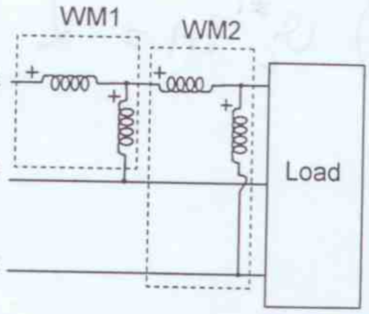
$$\vartheta_2(t) = |V_2|\cos(2\pi 50t + \measuredangle V_2)\ \text{Volts}_{RMS}.$$

Question 4 (10 pts)

In a balanced three-phase load with a positive phase sequence, the complex power is:

$$S = 1500 + j500\sqrt{3}\ \text{VA}$$

What are the wattmeter readings?

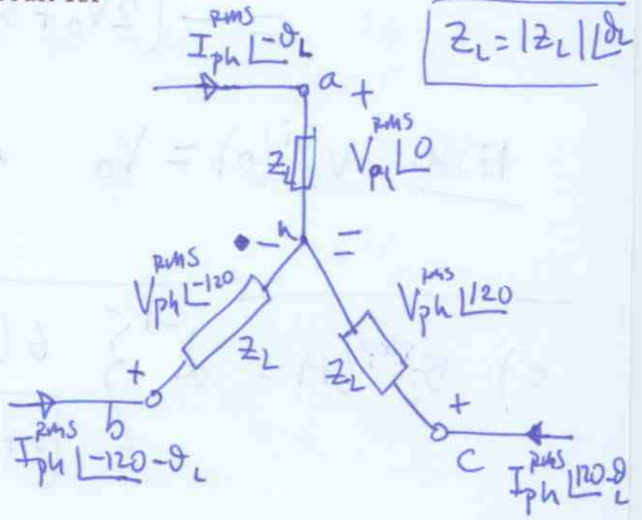


NOTE: You have to show your derivations to obtain credit for this question.

$$\boxed{Z_L = |Z_L| \angle \theta_L}$$

$$WM_1 \Rightarrow Re\{V_{ab} I_a^*\}$$

$$WM_2 \Rightarrow Re\{V_{ac} I_a^*\}$$

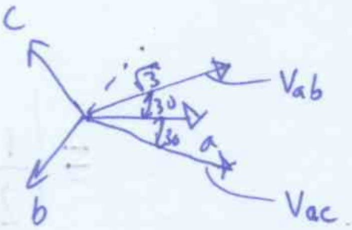


$$WM_1 \Rightarrow Re\{\sqrt{3} V_{ph}^{RMS} \angle 30 \ I_{ph}^{RMS} \angle +\theta_L\}$$

$$= \sqrt{3} V_{ph}^{RMS} I_{ph}^{RMS} \cos(30 + \theta_L)$$

$$= \sqrt{3} V_{ph}^{RMS} I_{ph}^{RMS} \left[\cos 30 \cos\theta_L - \sin 30 \sin\theta_L\right]$$

$$= \sqrt{3} V_{ph}^{RMS} I_{ph}^{RMS} \frac{\sqrt{3}}{2} \cos\theta_L - \sqrt{3} V_{ph}^{RMS} I_{ph}^{RMS} \frac{1}{2} \sin\theta_L$$



$$= \frac{3 V_{ph}^{RMS} I_{ph}^{RMS} \cos\theta_L}{2} - \frac{3 V_{ph}^{RMS} I_{ph}^{RMS} \sin\theta_L}{\sqrt{3} \cdot 2}$$

$$= \frac{1500}{2} - \frac{500\sqrt{3}}{\sqrt{3} \cdot 2}$$

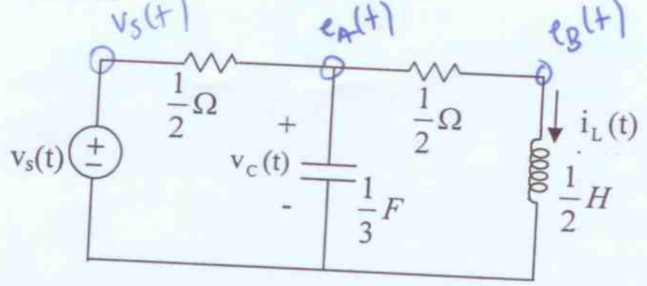
$$= 500 \text{ Watt.}$$

$$WM_2 \Rightarrow Re\{\sqrt{3} V_{ph}^{RMS} \angle -30 \ I_{ph}^{RMS} \angle \theta_L\}$$

$$= \sqrt{3} V_{ph}^{RMS} I_{ph}^{RMS} \cos(-30 + \theta_L)$$

$$= \frac{1500}{2} + \frac{500\sqrt{3}}{\sqrt{3} \cdot 2} = 1000 \text{ Watt.}$$

Question 5 (24 pts)



$v_C(0^-) = V_0$

$i_L(0^-) = I_0$

+4 **a)** Obtain the node equations in time domain, and put them in matrix form.

+4 **b)** Transform the node equation in part **(a)** to the s-domain.

+4 **c)** Transform the circuit to the s-domain, and then obtain the node equations in matrix form.

+4 **d)** Express $V_C(s)$ in terms of $V_s(s)$ and V_0 and I_0.

+4 **e)** Find the zero-input response for $v_C(t)$.

+4 **f)** Find the unit step response for $v_C(t)$.

a) KCL at $e_A \rightarrow (e_A - v_s)2 + \frac{1}{3}\dot{e}_A + (e_A - e_B)2 = 0$

KCL at $e_B \rightarrow (e_B - e_A)2 + I_0 + 2D^{-1}\{e_B\} = 0.$

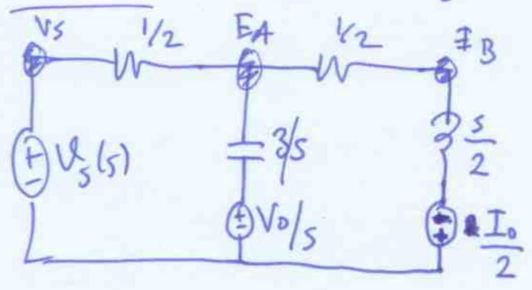
$$\begin{bmatrix} 4+\frac{p}{3} & -2 \\ -2 & 2+2D^{-1} \end{bmatrix} \begin{bmatrix} e_A \\ e_B \end{bmatrix} = \begin{bmatrix} 2v_s \\ -I_0 \end{bmatrix}$$

b) $(E_A - V_s)2 + \frac{1}{3}(sE_A - V_0) + (E_A - E_B)2 = 0$

$(E_B - E_A)2 + \frac{I_0}{s} + 2\frac{E_B}{s} = 0$

$$\begin{bmatrix} 4+\frac{s}{3} & -2 \\ -2 & 2+\frac{2}{s} \end{bmatrix} \begin{bmatrix} E_A(s) \\ E_B(s) \end{bmatrix} = \begin{bmatrix} 2V_s(s) + V_0/3 \\ -\frac{I_0}{s} \end{bmatrix}$$

c)



$$\begin{bmatrix} 2+2+\frac{s}{3} & -2 \\ -2 & 2+\frac{2}{s} \end{bmatrix} \begin{bmatrix} E_A(s) \\ E_B(s) \end{bmatrix} = \begin{bmatrix} \frac{V_0}{3} + 2V_s(s) \\ -\frac{I_0}{s} \end{bmatrix}$$

d) $V_C(s) = E_A(s) \rightarrow E_A(s) = \frac{1}{\Delta}\left[2+\frac{2}{s} \mid 2\right] \begin{bmatrix} \frac{V_0}{3} + 2V_s(s) \\ -I_0/s \end{bmatrix}$

$\Delta = \left(4+\frac{s}{3}\right)\left(1+\frac{1}{s}\right).2 - 4$

$= \frac{2s}{3} + \frac{14}{3} + \frac{8}{s} = \frac{2}{3s}(s^2+7s+12) = \frac{2}{3s}(s+4)(s+3)$

$E_A(s) = \left[\cancel{1+\frac{1}{s}}^{\,s+1} \mid \cancel{1}^{\,s}\right] \begin{bmatrix} V_0 + 6V_s(s) \\ -3I_0/s \end{bmatrix} \cdot \frac{s}{(s+4)(s+3)}$

$= \frac{(s+1)(V_0 + 6V_s(s)) - 3I_0}{(s+4)(s+3)} = \frac{(s+1)6V_s(s)}{(s+4)(s+3)} + \frac{(s+1)V_0 - 3I_0}{(s+4)(s+3)}$

d) $v_c^{zi}(t) = \mathcal{L}^{-1}\left\{ \dfrac{(s+1)V_0 - 3I_0}{(s+4)(s+3)} \right\}$

$$\frac{-2V_0-3I_0}{s+3} + \frac{3V_0+3I_0}{s+4}$$

$$= -(2V_0+3I_0)e^{-3t} + 3(V_0+I_0)e^{-4t}\ \text{V}. \quad t \geq 0$$

Note: $V_c^{zi}(0) = V_0$ as expected. (Also $\lim_{s\to\infty} sV_c^{zi}(s) = V_0$!)

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e) $v_c^{step}(t) = \mathcal{L}^{-1}\left\{ \dfrac{6(s+1)\,1/s}{(s+4)(s+3)} \right\}$

$$\to \frac{1/2}{s} + \frac{-4.5}{s+4} + \frac{4}{s+3}$$

$$= \left(\frac{1}{2} - 4.5e^{-4t} + 4e^{-3t}\right)u(t).$$

Note: $v_c^{step}(t) = 0$ (as expected)

$v_c^{step}(\infty) = 1/2$ (from circuit)

$h(0^+) = 6V$ (from circuit, impulse response at $t=0^+ \to V_c(0^+) = 6$)

$$\to \left.\frac{dv_c^{step}}{dt}(t)\right|_{t=0^+} = 18 - 12 = 6V \quad \text{(matching also)}$$