Middle East Technical University

Department of Electrical and Electronics Engineering

EE 531 – HW #3

Due: Nov. 24, 2014 (in-class)

1. (Textbook) Exercise 2.10

2. (Ross, Intro. Prob. Models, 10th Edition) Let $\{N(t), t \geq 0\}$ be a Poisson process with rate λ. For $s<t$, find

a. $P\{N(t) > N(s)\}$,

b. $P\{N(s) = 0 \mid N(t) = 3\}$,

c. $E\{N(t) \mid N(s) = 4\}$,

d. $E\{N(s) \mid N(t) = 4\}$.

3. (Ross, Intro. Prob. Models, 10th Edition) Let X and Y be independent exponential random variables with respective rates λ and μ. Let $M = \min(X, Y)$. Find

a. $\mathrm{E}\{MX \mid M = X\}$,

b. $\mathrm{E}\{MX \mid M = Y\}$,

c. $\mathrm{Cov}(X, M)$.

Hint: Consider modeling X and Y as two processes generated from a mother Poisson process.

4. (Ross, Intro. Prob. Models, 10th Edition) Cars pass a certain street location according to a Poisson process with rate λ. A woman who wants to cross the street at that location waits until she can see that no cars will come by in the next T time units.

a. Find the probability that her waiting time is 0.

b. Find her expected waiting time.

Hint: Check page 95 of textbook for the definition of X^*.

5. (Textbook) Exercise 2.12

6. (Textbook) Exercise 2.23 parts a, b and c.

7. (Ross, Intro. Prob. Models, 10th Edition) Let $\{N(t), t \geq 0\}$ be a non-homogeneous Poisson process with mean value function $m(t) = \int_0^t \lambda(t')dt'$. Given $N(t) = n$, show that the unordered set of arrival times has the same distribution as n independent and identically distributed random variables having the distribution function

$$F(x) = \begin{cases} \dfrac{m(x)}{m(t)} & x \leq t \\ 1 & x > t \end{cases}.$$

Hint: Extend the proof given for the homogeneous process.