

EE 504

Homework #3

Due: March 31, 2004

P.1: (Hayes Prob. 7.1) A random process $x[n]$ is generated as follows

$$x[n] = \alpha x[n-1] + v[n] + \beta v[n-1]$$

where $v[n]$ is white noise with mean μ_v and variance σ_v^2 .

a) Design a first-order linear predictor

$$\hat{x}[n+1] = w(0)x[n] + w(1)x[n-1]$$

that minimizes the mean-square error in the prediction of $x[n+1]$ and find the minimum MSE.

b) Now consider a predictor of the form

$$\hat{x}[n+1] = c + w(0)x[n] + w(1)x[n-1]$$

Find the values for $c, w(0), w(1)$ that minimizes the MSE and compare the MSE with that found in part (a).

P.2: (Hayes Prob. 7.9) We would like to estimate a process $d[n]$ from noisy observations

$$x[n] = d[n] + v[n]$$

where $v[n]$ is white noise with variance $\sigma_v^2 = 1$ and $d[n]$ is a wide-sense stationary random process with the first four values of the autocorrelation sequence given by

$$\mathbf{r}_d = [1.5, 0, 1.0, 0]^T \tag{1}$$

Assume that $d[n]$ and $v[n]$ are uncorrelated. Our goal is to design an FIR filter to reduce the noise in $x[n]$. Hardware constraints, however, limit the filter to only three nonzero coefficients in $W(z)$.

a) Derive the optimal three-multiplier filter

$$W(z) = w(0) + w(1)z^{-1} + w(2)z^{-2}$$

for estimating $d(n)$ and evaluate the MSE.

b) Repeat part (a) for the non-causal filter FIR filter

$$W(z) = w(-1)z + w(0) + w(1)z^{-1}$$

c) Can you suggest a way to reduce the MSE below that obtained for the filters designed in parts (a) and (b) without using any more than three coefficients ?

P.3: (Hayes Prob. 7.12) We observe a signal, $y[n]$, in a noisy and reverberant environment

$$y[n] = x[n] + 0.8x[n-1] + v[n]$$

where $v[n]$ is white noise with variance $\sigma_v^2 = 1$ and uncorrelated with $x[n]$. We know that $x[n]$ is a wide-sense stationary AR(1) random process with auto-correlation values: $\mathbf{r}_x = [4, 2, 1, 0.5]^T$.

- a) Find the non-causal IIR Wiener filter, $H(z)$, that produces the minimum MSE estimate of $x[n]$.
- b) Design a causal IIR Wiener filter that produces the minimum MSE estimate of $x[n]$.

P.4: (Haykin Prob. C.7.2) In this exercise, we look at the noise cancellation problem considered in Example Hayes 7.2.6 (also covered in class). Let

$$x[n] = d[n] + g[n]$$

where $d[n]$ is the harmonic process

$$d[n] = \sin(n\omega_0 + \phi)$$

with $\omega = 0.05\pi$ and ϕ is a random variable that is uniformly distributed between $-\pi$ and π . Assume that $g[n]$ is unit variance white noise. Suppose that a noise process $v_2[n]$ that is correlated with $g[n]$ is measured by a secondary sensor. The noise $v_2[n]$ is related to $g[n]$ by a filtering operation

$$v_2[n] = 0.8v_2[n] + g[n]$$

- a) Using MATLAB, generate 500 samples of the processes $x[n]$ and $v_2[n]$.
- b) Derive Wiener-Hopf equations that define the optimum p th-order FIR filter for estimating $g[n]$ from $v_2[n]$.
- c) Using filters of order $p = 2, 4$ and 6 , design and implement the Wiener noise cancellation filters. Make plots of the estimated process $\hat{d}[n]$ (noise removed sinusoid) and compare the average squared errors for each filters.
- d) In some situations, the desired signal may leak into the secondary sensor. In this case the performance of the Wiener filter may be severely compromised. To see what effect this has, suppose the input to the Wiener is:

$$v_0[n] = v_2[n] + \alpha d[n]$$

where $v_2[n]$ is the filtered noise defined above. Evaluate the performance of the Wiener noise canceller for several different values of α for filters of order $p = 2, 4$ and 6 . Comment on your observations.