EE 504

Homework #2
Due: March 17, 2004

P.1: (Hayes Prob. 3.9) Determine whether or not the following are valid autocorrelation matrices:

$$\mathbf{R}_1 = \begin{bmatrix} 4 & 1 & 1 \\ -1 & 4 & 1 \\ -1 & -1 & 4 \end{bmatrix}; \ \mathbf{R}_2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}; \ \mathbf{R}_3 = \begin{bmatrix} 1 & 1+j \\ 1-j & 1 \end{bmatrix}$$

$$\mathbf{R}_4 = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}; \ \mathbf{R}_5 = \begin{bmatrix} 2 & j & 1 \\ -j & 4j & -j \\ 1 & j & 2 \end{bmatrix}$$

P.2: (Hayes Prob. 3.10) The input to a LTI filter with impulse response

$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2)$$

is zero mean wide-sense stationary process with autocorrelation$r_x(k) = \left(\frac{1}{2}\right)^{|k|}$.

a) What is the variance of the output process ?
b) Find the autocorrelation of the output process, $r_y(k)$.

P.3: (Hayes Prob. 3.13) Suppose we are given a zero mean process $x(n)$ with autocorrelation

$$r_x(k) = 10\left(\frac{1}{2}\right)^{|k|} + 3\left(\frac{1}{2}\right)^{|k-1|} + 3\left(\frac{1}{2}\right)^{|k+1|}$$

a) Find a filter which, when driven by unit variance white noise, will yield a random process with this autocorrelation.
b) Find a stable and causal filter which, when excited by $x(n)$, will produce zero mean, unit variance white noise.

P.4: (Haykin Prob. 2.21) Consider an autoregressive process $u(n)$ of order 2, described with the following relation: $u(n) = u(n-1) - 0.5u(n-2) + v(n)$, where $v(n)$ is a white noise of zero mean and variance 0.5

a) Write Yule-Walker equations for the process.
b) Solve these two equations for the autocorrelation functions value $r_u(1)$ and $r_u(2)$.
c) Find the variance of $u(n)$.