EE 504

Homework #1
Due: March 10, 2004

P.1: Show that if $\rho_{xy} = 1$, then $y = ax + b$.

P.2: Work out the details of the following non-linear estimation problem. Assume that the observation x on y can take four different values, $x \in \{1, 2, 3, 4\}$. Show that the optimal non-linear estimator which minimizes the error $E\{(y - c(x))^2\}$ is $c(x) = E\{y|x\}$. Hint: Find c(1),c(2),c(3),c(4) such that

$$\begin{aligned} E\{(y - c(x))^2\} &= p\{x = 1\}E\{(y - c(1))^2|x = 1\} + \cdots \\ &\quad \cdots + p\{x = 4\}E\{(y - c(4))^2|x = 4\} \end{aligned}$$

is minimized.

P.3: Show that the optimal estimator for the Gaussian r.v.'s with Gaussian distributed observations is the linear estimator.

The pdf of the jointly distributed Gaussian r.v. (zero mean):

$$f(x, y) = \frac{1}{2\pi\rho_1\rho_2\sqrt{1 - r^2}} \exp\left[-\frac{1}{2(1 - r^2)}\left(\frac{x^2}{\rho_1^2} - 2r\frac{xy}{\rho_1\rho_2} + \frac{y^2}{\rho_2^2}\right)\right]$$

Show that $E\{y|x\}$ is a straight line. Assume non-zero mean. (Hint: See Papoulis page 176)

P.4: Modify the cost function of the estimation problem from $E\{(y - \hat{y})^2\}$ to $E\{|y - \hat{y}|\}$ and show that the minimum absolute error zeroth order estimation $\hat{y}$ is the median of the r.v. y. (Hint: See Papoulis page 178)

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METU, Spring 2004.
cc.