Identifying the joint dynamics of a bolted beam system by using FRF decoupling and the SEMM expansion technique

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ABSTRACT

This work extendes the FRF decoupling method to identify the dynamic effects of a bolted joint on a simple beam by using the SEMM expansion technique. The method removes the dynamic effects of the separate -and known- beam elements from measurements performed on the coupled system. This results in isolated effects that can be attributed to the joint. Additionally, as long as the basic properties of the joint remain constant (e.g. bolt length) one can assume that these isolated joint dynamics are independent from the structures themselves and thus system independent. This hypothesis is tested by coupling the newly identified bolted joint to dynamically different substructures in order to successfully predict the new overall system dynamics. This proves that (linear) joint dynamics can be identified and modelled with this method, and that, if the joint is sufficiently similar, this joint model is transferable to new systems.

Keywords: System Equivalent Model Mixing, Interfaces, Joint identification, Bolted Joints, Beams.

1 INTRODUCTION

The joints that connect structures influence the dynamic properties of the system. The influence is hard to predictively model since the precise mechanisms that cause it are not fully understood. However, measurements performed on the full system observe both the dynamic influence of the structures and the joint connecting them. If the dynamics of the structures can be separated from the dynamics of the joint with frequency based substructuring (FBS), we may identify the joint properties.

In the past, researchers have used a method based on FBS to successfully identified linear joint parameters of a bolted connection in a clamped beam [1]. It was limited to the identification of the joint in two degrees of freedom due to the physical limitation of the measurements. This work adds the system equivalent model mixing technique to the existing methodology in order to expand a measurement's DoF set. If the expansion is successful, the method may identify the joint identification in e.g. 6 degrees of freedom: 3 translations and 3 rotations.

The methods are based on the linear time-invariant assumption, and so any parameters that are identified are linearised parameters. Of course, friction based joints such as a bolted-joint are inherently non-linear, yet we assume that the linearised parameters may be valid.

2 THE JOINT IDENTIFICATION METHOD

2.1 Frequency Based Substructuring: Coupling and Decoupling with a weak-formulated joint

There are several ways to model the joint dynamics in a structure using a frequency based model. One method includes the joint as a separate substructure in the model. It assumes that the joint has all the characteristics of a normal substructure including mass or other internal dynamics. However, if the joint is self-equilibrating[1] another method is applicable. This method will be used here. What follows is a condensed explanation of frequency based dynamic substructring. For a more detailed explanation we suggest [2, 3] for the basic theory.

The equation of motion for the system A-B is:

$$\mathbf{u} = \mathbf{Y}(\mathbf{f}+\mathbf{g}) \quad \text{where:} \quad \mathbf{Y} = \begin{bmatrix} \mathbf{Y}^{A}_{ii} & \mathbf{Y}^{A}_{ib} & & \\ \mathbf{Y}^{A}_{bi} & \mathbf{Y}^{A}_{bb} & & \\ & & \mathbf{Y}^{B}_{bb} & \mathbf{Y}^{B}_{bi} \\ & & \mathbf{Y}^{B}_{ib} & \mathbf{Y}^{B}_{ii} \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \mathbf{u}^{A}_{i} \\ \mathbf{u}^{A}_{b} \\ \mathbf{u}^{B}_{b} \\ \mathbf{u}^{B}_{i} \end{bmatrix}, \quad \mathbf{f} = \begin{bmatrix} \mathbf{f}^{A}_{i} \\ \mathbf{f}^{A}_{b} \\ \mathbf{f}^{B}_{b} \\ \mathbf{f}^{B}_{i} \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} \mathbf{g}^{A}_{i} \\ \mathbf{g}^{A}_{b} \\ \mathbf{g}^{B}_{b} \\ \mathbf{g}^{B}_{i} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{g}^{A}_{b} \\ \mathbf{g}^{B}_{b} \\ \mathbf{0} \end{bmatrix} \tag{1}$$

Where $\mathbf{u}$ are the responses, $\mathbf{Y}$ the receptance frequency response functions (FRF), $\mathbf{f}$ the external forces, and $\mathbf{g}$ the boundary forces. The explicit dependency on frequency is omitted for clarity. To solve this equation of motion two sets of conditions are required which are referred to as the compatibility and equilibrium condition. The equilibrium condition states that the forces are equal but opposite on the interface. It is:

$$\mathbf{B}^T\boldsymbol{\lambda} = -\mathbf{g}, \quad \text{where} \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & -\mathbf{I} \end{bmatrix}, \quad \text{and } \boldsymbol{\lambda} = -\mathbf{g}^{A}_{b} = \mathbf{g}^{B}_{b} \tag{2}$$

where $\mathbf{B}$ is a signed Boolean matrix and $\boldsymbol{\lambda}$ is a Lagrange multiplier used to represent the interface forces. The compatibility condition states that the displacements on either side of the interface are equal but this is slightly altered when including a joint in the formulation [2]. It now is:

$$\mathbf{u}^{A}_{b} - \mathbf{u}^{B}_{b} = \Delta\mathbf{u} \quad \rightarrow \quad \mathbf{B}\mathbf{u} = \Delta\mathbf{u}, \quad \text{where} \quad \Delta\mathbf{u} = \mathbf{Y}^{J}\boldsymbol{\lambda} \tag{3}$$

Inserting the equilibrium and compatibility condition in the equation of motion results in the coupled system dynamics:

$$\mathbf{u} = \mathbf{Y}^{AB}\mathbf{f}, \quad \text{where} \quad \mathbf{Y}^{AB} = \mathbf{Y} - \mathbf{Y}\mathbf{B}^T\left(\mathbf{B}\mathbf{Y}\mathbf{B}^T + \mathbf{Y}^{J}\right)^{-1}\mathbf{B}\mathbf{Y} \tag{4}$$

Which is solved for the joint matrix $\mathbf{Y}^{J}$:

$$\mathbf{Y}^{J} = \mathbf{B}\mathbf{Y}\left(\mathbf{Y} - \mathbf{Y}^{AB}\right)^{-1}\mathbf{Y}\mathbf{B}^T - \mathbf{B}\mathbf{Y}\mathbf{B}^T \tag{5}$$

The attentive reader will be quick to remark that this is only possible if the matrices $\mathbf{BY}$ and $\mathbf{YB}^T$ are divisible is generally not true[3]. Note that the full system must have more or an equal number of DoF compared to the joint. If it is more, the joint may be identified in a least-squares sense. For a detailed alternative derivation of equation (5) from a transfer-function perspective the reader is suggested to read [1].

2.2 SEMM expansion

The DoF set of the uncoupled structures require multiple boundary DoF which may not be so straightforward to measure due to e.g. space restrictions. However, we can use System Equivalent Model Mixing (SEMM) to combine the measurements with numerical models in order to extrapolate the dynamics to the boundary DoF. SEMM works on the basis of FBS and is therefore easy to combine in the existing methodology. The full workings of SEMM are described in detail in [4]. However, for this work all that we need is the equation for the SEMM model:

$$\mathbf{Y}^{s} = \mathbf{N}^{s} - \mathbf{N}^{s}\left(\mathbf{N}^{s}_{i*}\right)^{+}\left(\mathbf{N}^{s}_{ii} - \mathbf{E}^{s}_{ii}\right)\left(\mathbf{N}^{s}_{*i}\right)^{+}\mathbf{N}^{s} \tag{6}$$

where the full model of structure (s) is made up from numerical $\mathbf{N}^{s}$ and experimental $\mathbf{E}^{s}$ FRF counterparts. Note that the experimental parts only contain the measurements on the interface DoF $(\bullet)_i$ whereas the numerical counterpart has all the DoF $(\bullet)_*$. The notations $(\bullet)^+$ indicate a pseudo-inverse.

[1] It is self-equilibrated if the forces on the substructure's interfaces connected by the joint are in equilibrium on all DoF.

[2] since the joint allows for a gap to exist in otherwise collocated DoF

[3] In these cases a generalized inverse is inherently assumed.

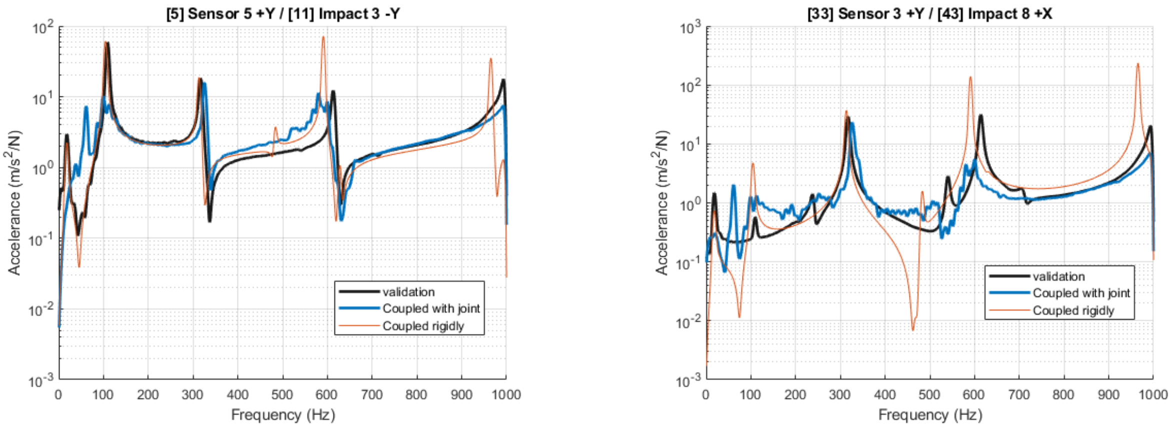
3 SETUP AND RESULTS

The system consists of two rectangle cross-section beams[4]. Part B is a 300 mm beam and Part A is a 450 mm beam. It is important that the dynamic contributions of the structures can change so that we may prove the identified joint is invariant to the system dynamics. This may classically be done by exchanging (one of) the components, but here it is done by changing the clamping location on Beam A. This has the added advantage that the joint properties, as well as the sensor locations remain equal.

(a) (b) (c)

Figure 1: The experimental set-up of the beam measurements. (a) the beam structure with part A (left) attached to part B (right). The picture also shows the different clamping locations. (b) a close-up of the bolted joint unattached. (c) The beam in the setup with one of the PCB tri-axial sensors.

The measurement campaign consists of 11 sensor channels (2 tri-axial and 5 single-axial sensors) and 16 dynamic hammer impact locations totalling to a 11×16 FRF matrix. These are the internal DoF that are measured both for the coupled configuration and the uncoupled configuration for Beam A. The uncoupled Beam B's model is fully substituted by a numerical model. The measurements of Beam A are expanded with 6 boundary DoF (3 translations and 3 rotations) using a numerical model according to the SEMM method described above and the virtual point method to obtain rotational DoF [5].



(a) (b)

Figure 2: Note that both FRF are smoothed to off-set the in-clarity due to noise.(a) The addition of the unfitted joint to the substructures does not massively improve the results, but does add noise. This is an FRF in the main deflection direction of the beam. (b) This is a cross-FRF between the main deflection direction and the direction along the length of the beam.

There are two measurement campaigns performed on the coupled beam, one for each clamping location (see Figure 1a). The measurements on the long-beam configuration will act as the validation measurement, while the measurements on the short-beam configuration are used to identify the joint. The identified joint is re-coupled to numerical models of the substructures (in the long-beam configuration) and validated. Figures 2a and 2b show the results of the experiments. These are the FRF of the validation measurement (black) vs. the FRF of the rigidly coupled (without joint) numerical models (red) and the FRF of the flexibly coupled (with the identified joint) models (blue).

[4]The beam cross section is 20 × 10 mm

While the effects of the joint are clearly seen, there seems to be no noticeable improvement relative to the rigid coupling with the exception of some frequencies. It is assumed this is due to the low coherence in the measurements of the short beam used to identify the joint. The measurements most probably contain some bias errors, e.g. double-impact errors[5]

4 DISCUSSION AND CONCLUSION

Unfortunately, even though the results are adequate in some directions, the overall result clearly demonstrates that the method was unsuccessful with these measurements since there is no improvement relative to the rigid coupling. The measurements will have to be redone in order to either validate or disprove the SEMM extended methodology.

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[5]unfortunately, the acquisition software did not have an automated double-impact detection, and the impact auto-power spectrum –which can be to detect the double impact– was not readily visible during the measurement. The effect of double-impacts was noticed in the short-beam FRF measurements.