First name:______________________________

Last name:______________________________

Student ID:______________________________

Signature:______________________________

Read before you start:

- There are five questions.
- The examination is open-book.
- Besides correctness, the CLARITY of your presentation will also be graded.

Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Total**

Q1.

Consider the following second-order system

$$\begin{aligned} \dot{x}_1 &= x_1^2 x_2 \\ \dot{x}_2 &= \sin(x_1) + u \end{aligned}$$

where u is the control input. Using backstepping, design a state feedback law $\phi : \mathbb{R}^2 \to \mathbb{R}$ such that the origin of the closed-loop system (obtained by letting $u = \phi(x_1, x_2)$) is globally asymptotically stable.

Q2.

(a) Using input-to-state stability (ISS) arguments, show that the origin of the below third-order system is globally asymptotically stable.

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2^2 + x_3^2 \\ \dot{x}_2 &= -x_2^3 + x_3^4 \\ \dot{x}_3 &= -x_3^5\end{aligned}$$

(b) Determine whether the below second-order system is ISS or not.

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{x_1}{1+|x_1|} - x_2 + u\end{aligned}$$

Q3.

Consider the following systems $\mathcal{H}_1$ and $\mathcal{H}_2$

$$\mathcal{H}_1 : \begin{cases} \dot{x} &= [S - \alpha C^T C]x + C^T e_1 \\ y_1 &= Cx\,, \end{cases} \qquad \mathcal{H}_2 : \begin{cases} \dot{z}_1 &= z_2^3 \\ \dot{z}_2 &= -\gamma z_1^3 - \beta z_2^3 + e_2 \\ y_2 &= z_2^3\,. \end{cases}$$

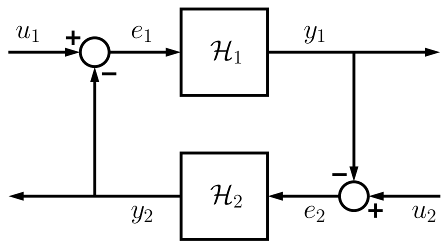
The system $\mathcal{H}_1$ is of order n, where $e_1 \in \mathbb{R}$ is the input, $y_1 \in \mathbb{R}$ is the output, $S \in \mathbb{R}^{n \times n}$ is a skew-symmetric matrix, $C \in \mathbb{R}^{1 \times n}$, and (the scalar) α is a positive constant. The system $\mathcal{H}_2$ is second order, whose input and output are e_2 and y_2, respectively. The constants β and γ are positive.

(a) Choosing a suitable storage function, show that the system $\mathcal{H}_1$ is output strictly passive. Find an upper bound on the $\mathcal{L}_2$ gain of the system.

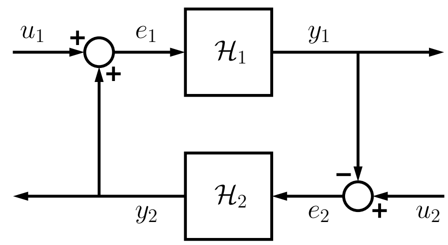
(b) Choosing a suitable storage function, show that the system $\mathcal{H}_2$ is also output strictly passive. Find an upper bound on the $\mathcal{L}_2$ gain of the system.

Q4.

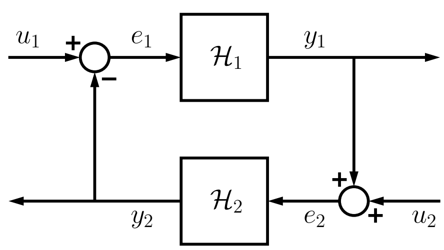
Consider the systems $\mathcal{H}_1$ and $\mathcal{H}_2$ of the previous problem. For each of the below feedback connections find (unrestrictive) conditions on the parameters α, β, γ under which the closed-loop system with input $u = [u_1\ u_2]^T$ and output $y = [y_1\ y_2]^T$ is finite-gain $\mathcal{L}_2$ stable.



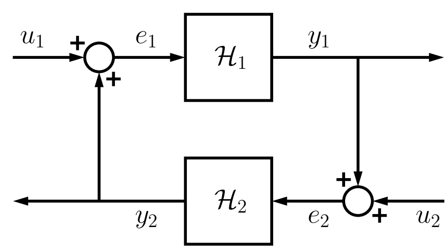
(a)



(b)



(c)



(d)

Q5.

Consider the feedback connection (c) of the previous problem under the condition that the external inputs are zero: $u_1 = u_2 = 0$. Let $\eta = [x^T \ z_1 \ z_2]^T \in \mathbb{R}^{n+2}$ denote the state of the closed-loop system. Investigate the stability properties of the origin $\eta = 0$ in terms of the parameters S, C, α, β, γ.