First name:____________________________

Last name:____________________________

Student ID:____________________________

Signature:____________________________

Read before you start:

- There are four questions.
- The examination is closed-book.
- No computer/calculator is allowed.
- The duration of the examination is 100 minutes.
- Please EXPLAIN all your answers.

Q1	**Q2**	**Q3**	**Q4**	**Total**

Q1.

It is known that the origin of $\dot{\eta} = [F + GH]\eta$ is asymptotically stable, where $F \in \mathbb{R}^{n \times n}$, $G \in \mathbb{R}^{n \times 1}$, and $H \in \mathbb{R}^{1 \times n}$. In particular, we have two symmetric positive definite matrices P, $Q \in \mathbb{R}^{n \times n}$ satisfying the Lyapunov equation

$$[F + GH]^T P + P[F + GH] = -Q\,.$$

Consider now the system

$$\begin{aligned} \dot{\eta} &= F\eta + G\xi \\ \dot{\xi} &= u\,. \end{aligned}$$

Using backstepping, design a feedback law $u = \psi(\eta,\, \xi)$ under which the origin of the closed-loop system is asymptotically stable.

Q2.

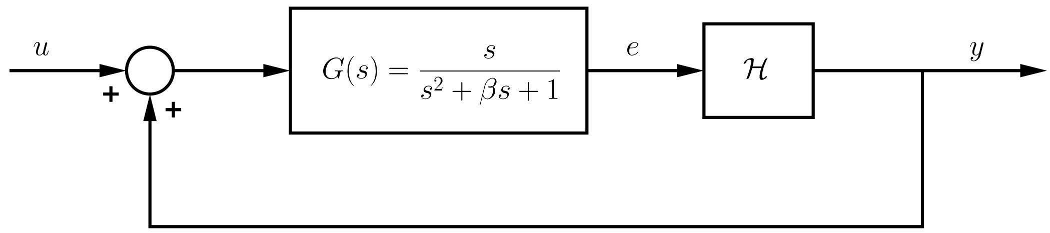
Consider the second order system

$$\mathcal{H}: \begin{cases} \dot{x}_1 &= -\alpha x_1^3 - x_2 + e \\ \dot{x}_2 &= x_1^3 \\ y &= x_1^3 \end{cases}$$

with $\alpha > 0$, where e is the input and y the output.

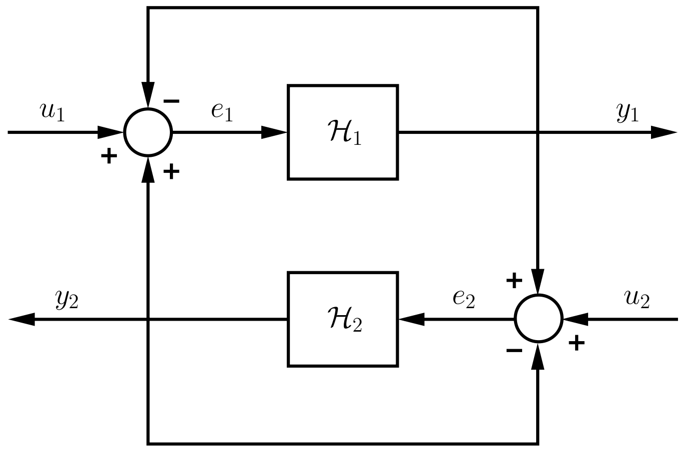
a) Show that the system $\mathcal{H}$ is finite-gain $\mathcal{L}_2$ stable. Find an upper bound on the gain. *Hint: You may want to work with* $V(x) = 0.25x_1^4 + 0.5x_2^2$.

b) Find a condition (in terms of α and β) under which the below closed-loop system (with input u and output y) is finite-gain $\mathcal{L}_2$ stable. Find an upper bound on the gain.



Q3.

Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two single-input single-output systems. Both $\mathcal{H}_1$ and $\mathcal{H}_2$ are output strictly passive and zero state observable. Consider the following feedback connection with input $u = [u_1\ u_2]^T$ and output $y = [y_1\ y_2]^T$.



a) Is the closed-loop system (from u to y) output strictly passive?

b) Is the closed-loop system zero state observable?

c) Is the origin of the unforced ($u = 0$) closed-loop system asymptotically stable?

Q4.

Determine whether each of the below systems is input-to-state stable or not. *Hint: If needed, you may want to work with* $V(x) = \|x\|^2$ *and use the inequality* $2(x_1^4 + x_2^4) \geq \|x\|^4$.

$$\textbf{a)} \begin{cases} \dot{x}_1 &= -x_1^3 + x_2 \\ \dot{x}_2 &= -x_1 - x_2^3 + u \end{cases} \qquad \textbf{b)} \begin{cases} \dot{x}_1 &= -x_1^3 - x_2 + u \\ \dot{x}_2 &= -x_1 - x_2^3 \end{cases}$$