First name:______________________________

Last name:______________________________

Student ID:______________________________

Signature:______________________________

Read before you start:

- There are four questions.
- The examination is closed-book.
- No computer/calculator is allowed.
- The duration of the examination is 100 minutes.
- Besides correctness, the CLARITY of your presentation will also be graded.

Q1	**Q2**	**Q3**	**Q4**	**Total**

Q1.

For each of the below dynamics determine whether the system has a periodic orbit.

(a) $$\begin{cases} \dot{x}_1 &= -x_2 + x_1(1 - 2x_1^2 - 3x_2^2) \\ \dot{x}_2 &= x_1 \end{cases}$$

(b) $$\begin{cases} \dot{x}_1 &= x_2 - x_1(1 - 2x_1^2 - 3x_2^2) \\ \dot{x}_2 &= -x_1 \end{cases}$$

Q2.

Consider the system

$$\begin{aligned}\dot{x}_1 &= (x_1x_2 - 1)x_1^3 + (x_1x_2 - 1 + x_2^2)x_1 \\ \dot{x}_2 &= -x_2\,.\end{aligned}$$

(a) Show that the set $\{x \in \mathbb{R}^2 : x_1x_2 \geq 2\}$ is forward invariant.

(b) Find all the equilibrium points of this system.

(c) Study the stability of each equilibrium point.

(d) Does this system have a GAS equilibrium point?

Q3.

Consider the second order system

$$\dot{x} = A\nabla V(x)$$

where $A \in \mathbb{R}^{2\times 2}$ and $V : \mathbb{R}^2 \to \mathbb{R}$ is a continuously differentiable, positive definite function whose gradient satisfies $\nabla V(x) \neq 0$ for all $x \neq 0$. For each of the following cases study the stability of the origin of this system.

(a) $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ **(b)** $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ **(c)** $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

Q4.

For each of the below systems study the stability of the origin.

(a) $\left\{\begin{array}{rcl}\dot{x}_1 & = & x_1 + x_2 \\ \dot{x}_2 & = & x_1^3 - x_2^3\end{array}\right.$ *Hint: Consider* $V(x) = \frac{1}{4}x_1^4 - \frac{1}{2}x_2^2$.

(b) $\left\{\begin{array}{rcl}\dot{x}_1 & = & -x_1 - x_2 \\ \dot{x}_2 & = & x_1^3\end{array}\right.$ *Hint: Consider* $V(x) = \frac{1}{4}x_1^4 + \frac{1}{2}x_2^2$.

(c) $\left\{\begin{array}{rcl}\dot{x}_1 & = & -x_1^3 - x_2 \\ \dot{x}_2 & = & x_1^3 - x_2|x_2|\end{array}\right.$ *Hint: Consider* $V(x) = \frac{1}{4}x_1^4 + \frac{1}{2}x_2^2$.