EE555 First Midterm Exam 16 Nov 2022

First name:_____

Last name:_____

Student ID:_____

Signature:_____

Read before you start:

- There are four questions.
- The examination is closed-book.
- No computer/calculator is allowed.
- The duration of the examination is 100 minutes.
- Besides correctness, the CLARITY of your presentation will also be graded.

Q1	Q2	Q3	Q4	Total

Q1.

For each of the below dynamics determine whether the system displays finite escape times.

(a)
$$\begin{cases} \dot{x}_1 &= -x_1 \\ \dot{x}_2 &= x_1 x_2^2 \end{cases}$$

(b)
$$\begin{cases} \dot{x}_1 &= x_1 \\ \dot{x}_2 &= x_1^2 x_2 \end{cases}$$

Q2.

Consider the second-order system

$$\dot{x} = [\, A - \|x\|^2 I \,]\, x$$

where $A \in \mathbb{R}^{2\times 2}$ is a nonsingular matrix.

(a) Find a general condition on A so that the system has a periodic orbit.

(b) Find a general condition on A so that the system has no periodic orbit.

Q3.

Consider the second-order system

$$\begin{aligned} \dot{x}_1 &= -ax_1 - 2x_2^3 \\ \dot{x}_2 &= x_1 - bx_2 \,. \end{aligned}$$

For each of the below cases determine if the origin is unstable, stable, asymptotically stable, globally asymptotically stable. (For some cases you may want to use $V(x) = x_1^2 + x_2^4$.)

(a) $a = 0$ and $b = 0$.

(b) $a = 1$ and $b = 1$.

(c) $a = 0$ and $b = -1$.

(d) $a = 1$ and $b = 0$.

Q4.

Consider the third-order system

$$\begin{aligned} \dot{x}_1 &= x_1^2 - x_1^3 \\ \dot{x}_2 &= x_1 x_3 - x_2 \\ \dot{x}_3 &= 1 - x_1 x_2 \,. \end{aligned}$$

(a) Find the (unique) equilibrium point of this system.

(b) Is the equilibrium point asymptotically stable? Explain.

(c) Is it globally asymptotically stable? Explain.