Problem 1 Exercise 4.54.

Problem 2 Consider the system below

$$\begin{aligned}\dot{x}_1 &= -ax_1 + x_2 \\ \dot{x}_2 &= bx_1 - g(x_1) - x_2 + u\end{aligned}$$

where $a > 0$, $g(0) = 0$, and $sg(s) \geq 0$ for all s. Using the below function

$$V(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + \int_0^{x_1} g(s)ds$$

find a condition on the parameter b so that the system is input-to-state stable.

Problem 3 Consider the below system

$$\dot{y} = f(y) + g(y)u$$

where f is continuously differentiable and has a bounded Jacobian matrix, g is continuous and bounded, and the origin of $\dot{y} = f(y)$ is globally exponentially stable. Using Theorem 4.14 show that the system is input-to-state stable with a linear gain function. [By gain function we mean the class-$\mathcal{K}$ function γ of the inequality (4.47) in the text.]

Problem 4 Consider the second-order system

$$\begin{aligned}\dot{x}_1 &= f(x_1) + g(x_2)x_2^4 \\ \dot{x}_2 &= -x_2|x_2|^3 + \varepsilon x_1\end{aligned}$$

where f and g satisfy the conditions of the previous problem. Show that the origin is globally asymptotically stable when $\varepsilon = 0$. Show also that if $\varepsilon \neq 0$ is sufficiently small we can still establish GAS. *Hint: You may want to use the Lyapunov function* $V(x) = V_1(x_1) + \frac{K}{5}|x_2|^5$ *where* $K > 0$ *is your design parameter and you already know what* V_1 *is from the previous problem.*

Problem 5 Exercise 5.10.

Problem 6 Exercise 5.13.

Problem 7 Exercise 5.15(4).

Problem 8 Exercise 5.16. *Hint: See the Lyapunov function construction of Problem 2.*