Problem 1 Consider the system $\dot{x} = f(x)$ with $f(0) = 0$. A solution $x(t)$ is known to satisfy $x(0) \neq 0$ and $x(T) = 0$ for some finite $T > 0$. Show that f does not satisfy Lipschitz condition at the origin, that is, no $L > 0$ and $\varepsilon > 0$ exist such that $\|f(x) - f(y)\| \leq L\|x - y\|$ for all $x, y \in \{\eta \in \mathbb{R}^n : \|\eta\| \leq \varepsilon\}$.

Problem 2 For each of the below systems determine whether or not finite escape times occur.

(a) $\begin{cases} \dot{x}_1 &= -x_1 + x_1^2 x_2 \\ \dot{x}_2 &= -x_2 \end{cases}$ **(b)** $\begin{cases} \dot{x}_1 &= x_2^2 x_1 \\ \dot{x}_2 &= x_2 + 10 \end{cases}$

Problem 3 Consider the system $\dot{x} = f(x)$ where f is continuous and $f(\lambda x) = \lambda f(x)$ for all $\lambda > 0$ and $x \in \mathbb{R}^n$.

(a) Show that if $\phi(t)$ is a solution to this system so is $\lambda\phi(t)$ for each $\lambda > 0$.

(b) Show that this system cannot exhibit finite escape times.

(c) Show that no solution $x(t)$ exists satisfying $x(0) \neq 0$ and $x(T) = 0$ for some finite $T > 0$.

Problem 4 Exercise 4.3.

Problem 5 Exercise 4.7.

Problem 6 Exercise 4.21.