First name:______________________________

Last name:______________________________

Student ID:______________________________

Signature:______________________________

Read before you start:

- There are four questions.
- The duration of the examination is 100 minutes.
- The examination is closed-book.
- Please explain all your answers.

Q1	**Q2**	**Q3**	**Q4**	**Total**

Q1.

Consider the following system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 5 & -3 & 3 & 1 \\ 6 & -4 & 1 & 4 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & -5 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} u \\ y &= [2 \ -1 \ 2 \ 7]\, x\,. \end{aligned}$$

(a) Is this system in controllable decomposition form?

(b) Is this system stabilizable?

(c) Is this system BIBO stable?

(d) Determine, if possible, the range of $k \in \mathbb{R}$ for which the output feedback law $u = -ky$ renders the closed-loop system asymptotically stable.

Q2.

Given

$$A = \begin{bmatrix} 0 & 1 \\ -0.5 & -1.5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

determine (without direct computation) whether a symmetric positive definite solution $P \in \mathbb{R}^{2\times 2}$ exists or not for each of the following matrix (in)equalities.

(a) $AP + PA^T + BB^T = 0.$

(b) $A^T P + PA + BB^T = 0.$

(c) $APA^T - P - BB^T < 0.$

(d) $A^T PA - P - BB^T < 0.$

Q3.

Consider the system

$$\dot{x} = \begin{bmatrix} 0 & 4 \\ 2 & 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u\,.$$

(a) Is this system stable?

(b) Is this system controllable?

(c) Is this system stabilizable?

(d) Find, if possible, a feedback gain $K \in \mathbb{R}^{1\times 2}$ such that the feedback law $u = -Kx$ renders the closed-loop system asymptotically stable.

Q4.

Definition. Let $\beta_i : \mathbb{R} \to \mathbb{R}$ for $i = 1, 2, \ldots, n$ be given. The functions $\beta_1(t), \beta_2(t), \ldots, \beta_n(t)$ are said to be *linearly independent* if the equation

$$\alpha_1\beta_1(t) + \alpha_2\beta_2(t) + \cdots + \alpha_n\beta_n(t) = 0 \qquad \forall t \in \mathbb{R}$$

(where $\alpha_1, \alpha_2, \ldots, \alpha_n \in \mathbb{R}$) has no solution other than the trivial solution: $\alpha_i = 0$ for all i.

Now, prove the following claim.

Claim. Let the system $\dot{x} = Ax + Bu$ (with $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times 1}$) be controllable. Then the entries of the vector $e^{At}B$ must be linearly independent.