First name:______________________________

Last name:______________________________

Student ID:______________________________

Signature:______________________________

Read before you start:

- There are four questions.
- The examination is open-book.
- No calculator is allowed.
- The duration of the examination is 100 minutes.
- PLEASE EXPLAIN ALL YOUR ANSWERS.

Q1	**Q2**	**Q3**	**Q4**	**Total**

Q1.

Consider the below system.

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -4 & 3 & 1 \\ -6 & 5 & 0 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u \\ y &= [-2 \quad 1 \quad 7]\, x\,. \end{aligned}$$

(a) Is this system stable in the sense of Lyapunov?

(b) Is this system controllable?

(c) Is this system stabilizable?

(d) Is this system BIBO stable?

Q2.

Consider the following third order system with scalar input and scalar output

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= [\alpha \;\; \beta \;\; 2]\, x \end{aligned}$$

whose full-state impulse response, i.e., $h(t) = \mathcal{L}^{-1}\{(sI - A)^{-1}B\}$ is given below.

$$h(t) = \begin{bmatrix} e^t + e^{-t} \\ 2 + e^{-t} \\ 1 - e^t \end{bmatrix}$$

(a) Find B.

(b) Compute the controllability matrix.

(c) Does the vector $\bar{x} = [1 \;\; 0 \;\; 1]^T$ belong to the reachable subspace?

(d) Find α and β so that the system is BIBO stable.

Q3.

Consider the below system which is known to be uncontrollable but stabilizable.

$$\dot{x} = \begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix} x + \begin{bmatrix} 2 \\ \alpha \end{bmatrix} u$$

(a) Find α.

(b) Obtain the controllable decomposition $\dot{z} = \bar{A}z + \bar{B}u$ via the transformation $z = T^{-1}x$, where the columns of T are the eigenvectors of A.

(c) Choose a diagonal $\bar{P} > 0$ satisfying the Lyapunov inequality $\bar{A}\bar{P} + \bar{P}\bar{A}^T - \bar{B}\bar{B}^T < 0$.

(d) Choose a gain $K \in \mathbb{R}^{1\times 2}$ in terms of $\bar{B}$, $\bar{P}$, T such that the feedback law $u = -Kx$ makes the closed-loop system exponentially stable. *Do not compute K here; just give its expression.*

Q4.

Consider the predator-prey equations

$$\begin{aligned}\dot{x}_1 &= x_1(1 - x_2)\\ \dot{x}_2 &= -x_2(1 - x_1)\,.\end{aligned}$$

(a) Find all the equilibrium points of this system.

(b) At each equilibrium obtain the linearization.

(c) What can (based on the linearization) be said about the local stability of each equilibrium?