First name:______________________________

Last name:______________________________

Student ID:______________________________

Signature:______________________________

Read before you start:

- There are four questions.
- The examination is open-book.
- No calculator is allowed.
- The duration of the examination is 100 minutes.
- PLEASE EXPLAIN ALL YOUR ANSWERS.

Q1	**Q2**	**Q3**	**Q4**	**Total**

Q1.

Let $P \in \mathbb{R}^{n \times n}$ be an orthogonal projection matrix, i.e., $P^2 = P$ and $P^T = P$. Suppose $P \neq I$. Let $v \in \mathbb{R}^n$ be a unit vector ($v^T v = 1$) that belongs to the range space of P. Now consider the single input single output system

$$\begin{aligned} \dot{x} &= Px + vu \\ y &= v^T x\,. \end{aligned}$$

(a) Write the minimal polynomial of P. *Hint:* $P^2 = P$.

(b) Show that $Pv = v$.

(c) Compute e^{Pt}. *Hint: use the power series definition.*

(d) Compute the transfer function $G(s) = Y(s)/U(s)$.

Answer:

Q2.

Let $P \in \mathbb{R}^{n \times n}$ be an orthogonal projection matrix, i.e., $P^2 = P$ and $P^T = P$. Suppose $P \neq I$. Let $v \in \mathbb{R}^n$ be a unit vector ($v^T v = 1$) that belongs to the range space of P. Again consider the single input single output system

$$\begin{aligned} \dot{x} &= Px + vu \\ y &= v^T x \,. \end{aligned}$$

(a) Is this system BIBO stable? Is it internally stable?

(b) Compute the controllable subspace $\mathcal{S}_\mathrm{c}$.

(c) Let $w \in \mathbb{R}^n$ be such that w are v are linearly independent. Can we find an input signal $u(t)$ that transfers the state $x(t)$ from the origin $x(0) = 0$ to $x(1) = w$?

(d) Is this system stabilizable?

Answer:

Q3.

Consider the system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 5 & -4 \\ 8 & -7 \end{bmatrix} x + \begin{bmatrix} \alpha \\ 1 \end{bmatrix} u \\ y &= [-1 \;\; \beta]\, x\,. \end{aligned}$$

(a) Is this system stable in the sense of Lyapunov?

(b) Find α so that the system is BIBO stable for all β.

(c) Is the system stabilizable for the α you found in part (b)?

(d) Find β so that the system is BIBO stable for all α.

Answer:

Q4.

Consider the single input single output system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

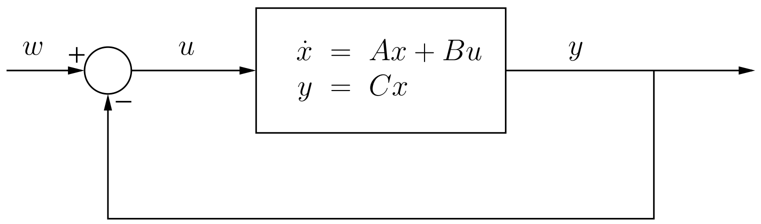
where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(a) Show that this system is controllable.

(b) Show that $\det(sI - A) = s^3 + cs^2 + bs + a$.

For the rest of the question consider the below feedback configuration.



$$\frac{Y(s)}{W(s)} = \frac{\cdots}{(s+1)^3}$$

(c) Find the state space representation of the closed-loop system. I.e., find the triple $(A_{\text{cl}}, B_{\text{cl}}, C_{\text{cl}})$ in terms of A, B, C such that $\dot{x} = A_{\text{cl}}x + B_{\text{cl}}w$ and $y = C_{\text{cl}}x$.

(d) Find C in terms of a, b, c.