First name:______________________________

Last name:______________________________

Student ID:______________________________

Signature:______________________________

Read before you start:

- There are four questions.
- The examination is closed-book.
- No calculator is allowed.
- The duration of the examination is 100 minutes.
- PLEASE EXPLAIN ALL YOUR ANSWERS.

Q1	**Q2**	**Q3**	**Q4**	**Total**

Q1.

Consider the first-order LTV system

$$\dot{x} = A(t)x\,, \qquad t \geq 0$$

a solution of which is known to be $x(t) = \dfrac{1}{1+t}$.

(a) Find $A(t)$.

(b) Find the state transition matrix $\Phi(t,\, t_0)$.

(c) Find the solution starting from the initial condition $x(3) = -7$.

(d) Is this system stable? Is it asymptotically stable? Is it exponentially stable?

Q2.

The impulse response of a single input single output LTI system reads $h(t) = 1 + te^{-3t}$.

(a) Obtain the transfer function $H(s)$ of this system.

(b) Write a state space representation $\left\{ \begin{array}{rcl} \dot{x} & = & Ax + Bu \\ y & = & Cx \end{array} \right\}$ for this system.

(c) Write the Jordan form of A.

(d) Find an initial condition $x(0)$ which yields $y(t) = h(t)$ for zero input $u(t) \equiv 0$.

Q3.

Let a matrix $A \in \mathbb{R}^{n \times n}$ satisfy

$$A^T P A - \rho^2 P < 0$$

for some symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$ and positive real number ρ.

(a) Show that all eigenvalues of A satisfy $|\lambda_i| < \rho$.

(b) Let $x(k)$ denote the solution of the system $x^+ = Ax$. Show that we can find some $c > 0$ such that $\|x(k)\| \leq c\rho^k \|x(0)\|$ for $k = 0, 1, 2, \ldots$

Q4.

Consider

$$
\begin{aligned}
\dot{x} &= \begin{bmatrix} -6 & a_{12} \\ a_{21} & a_{22} \end{bmatrix} x \\
y &= [4 \;\; c_2]\, x\,.
\end{aligned}
$$

It is known that $[1 \;\; 2]^T$ is an equilibrium point for this system. Moreover, it has been observed that for all initial conditions $x(0) = x_0$ the output always has the form $y(t) = ke^{-t}$, where the constant k depends on x_0.

(a) Find the eigenvectors of the system matrix A.

(b) Determine $\operatorname{null} A$ and $\operatorname{range} A$.

(c) Find c_2.

(d) Determine the stability properties of the system.