First name:____________________________

Last name:____________________________

Student ID:____________________________

Did you take EE501? ☐ **Yes** ☐ **No**

Signature:____________________________

Read before you start:

- There are four questions.
- The examination is closed-book.
- No calculator is allowed.
- The duration of the examination is 100 minutes.
- PLEASE EXPLAIN ALL YOUR ANSWERS.

Q1	**Q2**	**Q3**	**Q4**	**Total**

Q1.

Given two nonzero vectors B, $C^T \in \mathbb{R}^n$ consider the system

$$\begin{aligned} \dot{x} &= BCx + Bu \\ y &= Cx\,. \end{aligned}$$

(a) Obtain the state transition matrix. Characterize stability in terms of $\lambda := CB$.

(b) Is this system zero state equivalent to $\left\{ \begin{array}{rcl} \dot{z} &=& CBz + CBu \\ y &=& z \end{array} \right\}$?

Q2.

For each of the below cases determine the stability of the system $x^+ = Ax$.

(a) $A^T A - I < 0\,.$

(b) $A^T A - I = 0\,.$

(c) $A^T A - I > 0\,.$

Q3.

For $A,\, B \in \mathbb{R}^{n\times n}$ consider the LTV system

$$\dot{x} = e^{-At} B e^{At} x \,.$$

(a) Obtain the state transition matrix $\Phi(t,\, \tau)$. *Hint: employ* $z(t) := e^{At}x(t)$.

(b) Find the solution $x(t)$ given that $Ax(0) = \alpha x(0)$ and $Bx(0) = \beta x(0)$ for some $\alpha,\, \beta \in \mathbb{R}$.

Q4.

For $y \in \mathbb{R}^n$ and $Q = Q^T > 0$ consider the system

$$\ddot{y} + Q^{-1} y = 0\,.$$

(a) Obtain a state space representation $\left\{ \begin{array}{rcl} \dot{x} & = & Ax \\ y & = & Cx \end{array} \right\}$.

(b) Find a block diagonal $P = P^T > 0$ that satisfies $A^T P + PA = 0$.

(c) Can A have a 2×2 Jordan block?