First name:____________________________

Last name:____________________________

Student ID:____________________________

Signature:____________________________

Read before you start:

- There are four questions.
- The duration of the examination is 120 minutes.
- The examination is closed-book.
- Please explain all your answers.

Q1	**Q2**	**Q3**	**Q4**	**Total**

Q1.

Consider the following single-input single-output system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix} x + Bu \\ y &= Cx\,. \end{aligned}$$

(a) Is this system internally stable?

(b) Find a $B \in \mathbb{R}^{2\times 1}$ such that the system is uncontrollable but stabilizable.

(c) Find a nonzero $C \in \mathbb{R}^{1\times 2}$ such that the system is undetectable.

(d) Let the B matrix be what you found in part (b). Find (if possible) a nonzero initial condition $x(0) \in \mathbb{R}^2$ for which there exists a control input $u(t)$ that achieves $x(1) = 0$.

Q2.

Consider the system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 2 & 4 \\ -2 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \,. \end{aligned}$$

(a) Is this system a minimal realization of its transfer function?

(b) Let $x(0) = 0$. Find (if possible) a bounded input signal $u(t)$ that results in an unbounded output $y(t)$.

(c) Find the feedback gain $K \in \mathbb{R}^{1\times 2}$ such that the state feedback $u = -Kx$ places the closed-loop system poles at $\lambda_{1,2} = -2 \pm j2$.

(d) Design an observer for this system (i.e., compute the observer gain $L \in \mathbb{R}^{2\times 1}$) such that the eigenvalues of the error dynamics are at $\lambda_{1,2} = -1, -2$. Write also the observer dynamics.

Q3.

Let

$$A = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}.$$

(a) Write the Jordan form of A. *Hint:* $A^2 = ?$

(b) Compute e^{At}.

(c) Determine the stability of the system $x^+ = Ax$.

(d) Find (if possible) a matrix $C \in \mathbb{R}^{1\times 3}$ such that the pair (C, A) is observable.

Q4.

Let $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$. Suppose $A = A^T > 0$ and the pair (A, B) is controllable. Consider now the system

$$\ddot{x} + BB^T\dot{x} + Ax = Bu\,.$$

(a) Show that this system is controllable.

(b) Show that this system is asymptotically stable. *Hint: Try* $P = \frac{1}{2}\begin{bmatrix} A^{-1} & 0 \\ 0 & I \end{bmatrix}$.