First name:____________________

Last name:____________________

Student ID:____________________

Signature:____________________

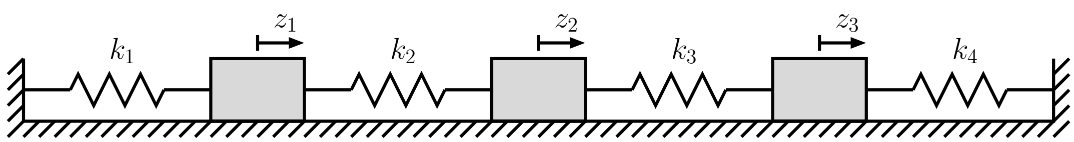
Read before you start:

- There are FIVE QUESTIONS.
- The examination is OPEN-BOOK.
- Besides correctness, the CLARITY of your presentation will also be graded.
- NO COLLABORATION with others.

Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Total**

Q1.

Consider the LTI mass spring system below, where there is no friction. The blocks are identical with unit mass, i.e., $m_1 = m_2 = m_3 = 1$. The displacements of the blocks from the equilibrium positions are denoted by z_1, z_2, $z_3 \in \mathbb{R}$ as shown in the figure. The spring constants are denoted by $k_i > 0$ for $i = 1, 2, 3, 4$. For each of the following cases find (if exists) a condition[1] on the spring constants so that the system is unobservable from the indicated output y.



(a) $y = z_1$.

(b) $y = z_2$.

[1] Your condition should be of the form $\sum_{i=1}^{4} \alpha_i k_i = 0$ for some $\alpha_i \in \mathbb{R}$.

Q2.

Let P, Q, $R \in \mathbb{R}^{n\times n}$ be symmetric positive definite matrices.

(a) Find (if possible) a pair (P, Q) such that the following system is unstable. Explain if impossible.

$$\dot{x} = \begin{bmatrix} 0 & -P \\ Q & 0 \end{bmatrix} x$$

(b) Find (if possible) a triple (P, Q, R) such that the following system is stable. Explain if impossible.

$$\dot{x} = \begin{bmatrix} P & -R \\ R & Q \end{bmatrix} x$$

Q3.

Let $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$. Consider the discrete-time system $x(k+1) = Ax(k) + Bu(k)$. Suppose this system is stabilizable. Hence we can find a feedback gain $K_1 \in \mathbb{R}^{m\times n}$ such that under the feedback law $u(k) = -K_1 x(k)$ the closed-loop system is exponentially stable. Does this imply that we can find a feedback gain $K_2 \in \mathbb{R}^{m\times n}$ such that the closed-loop system under $u(k) = -K_2 x(k-1)$ is exponentially stable? Explain.

Q4.

Consider the following system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 1 & 0 \\ -2 & 0 & 2 \\ 1 & 1 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ y &= [1 \quad -1 \quad 0]\, x\,. \end{aligned}$$

(a) Is this system detectable? Explain.

(b) Is this system BIBO stable? Explain.

Q5.

Consider the following system

$$
\begin{aligned}
x^{+} &= \begin{bmatrix} -1 & 1 & 0 \\ -2 & 0 & 2 \\ 1 & 1 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\
y &= [1 \;\; -1 \;\; 0]\, x\,.
\end{aligned}
$$

Design the observer gain $L \in \mathbb{R}^{3\times 1}$ and the feedback gain $K \in \mathbb{R}^{1\times 3}$ such that under the observer-based feedback law $u = -K\hat{x}$ (where $\hat{x}$ is the state estimate generated by your observer) the closed-loop system is finite-time stable. That is, there exists some finite integer N such that all the solutions satisfy $x(k) = 0$ for $k \geq N$ regardless of the initial conditions $x(0)$ and $\hat{x}(0)$. Write also your observer dynamics.