First name:____________________________

Last name:____________________________

Student ID:____________________________

Signature:____________________________

Read before you start:

- There are four questions.
- The examination is closed-book.
- No calculator is allowed.
- The duration of the examination is 150 minutes.
- PLEASE EXPLAIN ALL YOUR ANSWERS.

Q1	**Q2**	**Q3**	**Q4**	**Total**

Q1.

Consider the system below.

$$
\begin{aligned}
\dot{x} &= \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\
y &= \begin{bmatrix} 2 & 1 \end{bmatrix} x\,.
\end{aligned}
$$

a) Is this system internally stable?

b) Is this system BIBO stable?

c) Is this system observable?

d) Is this system detectable?

Q2.

Consider the following system with impulse response $h(t) = 1 + e^t$.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx\,. \end{aligned}$$

a) Find a suitable triple $(A,\ B,\ C)$ such that the system is a minimal realization of $H(s)$.

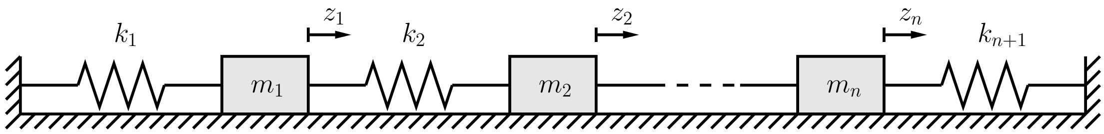
b) For your design in part (a) find a feedback gain K such that under the state feedback $u = -Kx$ all the closed-loop eigenvalues are located at $\lambda = -1$.

c) For your design in part (a) construct an observer such that the eigenvalues of the error dynamics are located at $\lambda = -1$.

d) Find a suitable triple $(A,\ B,\ C)$ that is a realization of $H(s)$ and satisfies all of the following constraints:

- A has an eigenvalue at $\lambda = -2$,
- The system is uncontrollable but stabilizable,
- The unobservable subspace contains the vector $[1\ \ 0\ \ 1]^T$.

Q3.



Consider the coupled mass spring systems above whose dynamics read $M\ddot{z} + C^T C\dot{z} + Kz = 0$, where $z = [z_1\ z_2\ \dots\ z_n]^T \in \mathbb{R}^n$ and the matrices $M,\ K \in \mathbb{R}^{n\times n}$ are symmetric positive definite. The viscous friction is represented by $C \in \mathbb{R}^{q\times n}$.

a) For the state choice $x = [x_1^T\ x_2^T]^T$ with $x_1 = z$ and $x_2 = \dot{z}$ obtain the state space representation $\dot{x} = Ax$.

b) Employing the matrix $P = \begin{bmatrix} K/2 & 0_{n\times n} \\ 0_{n\times n} & M/2 \end{bmatrix}$ show that this system is stable. *Note that $x^T P x$ is the total energy of the system.*

c) Show that if the pair $(C,\ M^{-1}K)$ is observable then so is the pair $([0_{q\times n}\ C],\ A)$.

d) Show that if the pair $(C,\ M^{-1}K)$ is observable then the system is exponentially stable.

Q4.

Let

$$A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$$

a) Find the minimal polynomial and eigenvector(s) of A.

b) Is the system $\dot{x} = Ax$ stable?

c) Is the system $x^+ = Ax$ stable?

d) Find (if possible) a symmetric positive definite matrix $P \in \mathbb{R}^{2\times 2}$ satisfying $A^T PA - P < 0$.