First name:____________________________

Last name:____________________________

Student ID:____________________________

Signature:____________________________

Read before you start:

- There are four questions.
- The examination is open-book.
- No computer/calculator is allowed.
- The duration of the examination is 150 minutes.
- PLEASE EXPLAIN ALL YOUR ANSWERS.

Q1	**Q2**	**Q3**	**Q4**	**Total**

Q1.

Consider the following system.

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 3 & -4 & 0 \\ 2 & -3 & 0 \\ 1 & 0 & 2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} u \\ y &= [0 \;\; 1 \;\; 0]\, x\,. \end{aligned}$$

(a) Is this system controllable?

(b) Is this system observable?

(c) Can we stabilize this system by state feedback $u = -Kx$?

(d) Can we asymptotically estimate the state of this system by an observer?

(e) Is this system BIBO stable?

Q2.

Consider the following system.

$$\begin{aligned} \dot{x} &= \begin{bmatrix} \alpha & 8 \\ -4 & \beta \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u \\ y &= [4 \quad -1]\, x\,. \end{aligned}$$

It is known that this system is observable and BIBO stable. Also, for some initial condition $x(0) = \eta$ and zero input $u = 0$, the output is measured to be $y(t) = 9e^t$.

(a) Find α and β.

(b) Find η.

Q3.

Let $\{v_1, v_2, \ldots, v_n\}$ be an orthonormal basis for $\mathbb{R}^n$, that is, $v_i^T v_i = 1$ and $v_i^T v_j = 0$ for $i \neq j$. Define $A := v_1 v_2^T + v_2 v_1^T$.

(a) Write range(A).

(b) Write null(A).

(c) Determine the minimal polynomial of A. *Hint:* $A^3 = ?$

(d) Find all the eigenvalues of A.

(e) Find all the eigenvectors of A.

(f) Write the Jordan form of A.

(g) Compute e^{At}.

(h) Compute A^k.

(i) Determine the stability of $\dot{x} = Ax$.

(j) Determine the stability of $x^+ = Ax$.

Q4.

Theorem. *Let A, $P \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{m\times n}$. Suppose (C, A) is observable, $P^T = P > 0$, and the Lyapunov equality $A^T PA - P = -C^T C$ holds. Then the system $x^+ = Ax$ is exp. stable.*

Proof. ?