Phasor Domain Analysis

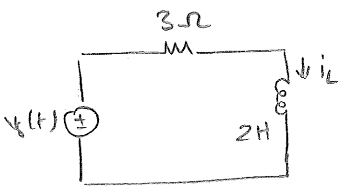
[Goal : Solve the circuit in sinusoidal steady state.]

Step1 Transform the circuit into phasor domain by representing the input and circuit variables as phasors and passive circuit elements by their impedances.

Step2 Solve the phasor domain circuit treating it like an LTI resistive circuit. That is, obtain the phasors of the asked variables.

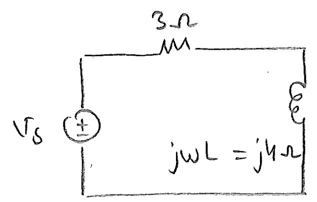
Step3 Transform the phasors (obtained in step 2) into their time domain representations.

Example (Revisited) Find $i_L(t)$ in steady state.



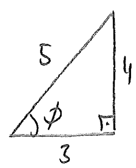
$v_s(t) = 10\cos 2t$ V

Sol'n Phasor domain circuit: ($\omega = 2$ rad/sec)



$V_s = 10\angle 0^\circ = 10$ V

$$I_L = \frac{V_s}{3+j4} = \frac{10}{5\angle\phi} = 2\angle{-\phi} \text{ A} \quad (1)$$

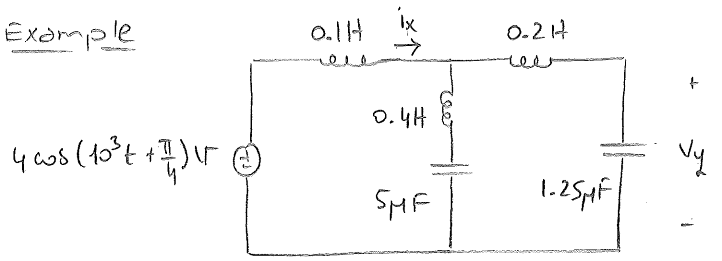


(1) $\Rightarrow$ $i_L(t) = 2\cos(2t - \phi) = \boxed{2\cos\left(2t - \arctan\frac{4}{3}\right) \text{ A}}$

$$= 2\{\cos\phi\cos 2t + \sin\phi\sin 2t\} = 2\{0.6\cos 2t + 0.8\sin 2t\}$$

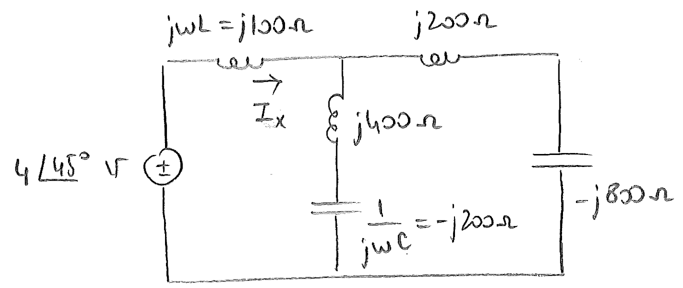
$$= 1.2\cos 2t + 1.6\sin 2t \text{ A}$$

Example



Find $i_x(t)$ & $v_y(t)$ in steady state.

Sol'n Equivalent circuit in phasor domain at freq. $\omega = 10^3$ rad/sec:



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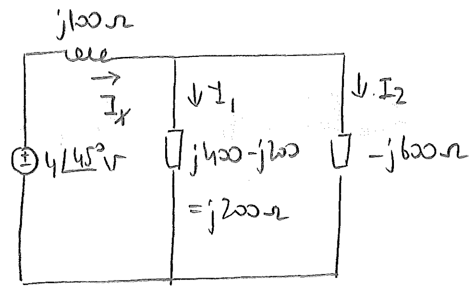

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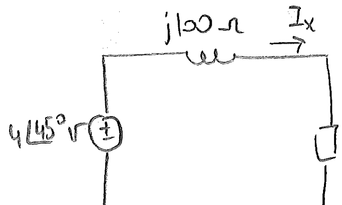


$$Z = \left\{ \frac{1}{j200} + \frac{1}{-j600} \right\}^{-1} = \frac{j200(-j600)}{j200 - j600} = \frac{200 \times 600}{-j400} = j300\,\Omega$$

$$\Rightarrow I_x = \frac{4\angle 45°}{j100 + j300} = \frac{4\angle 45°}{400\angle 90°} = \frac{1}{100}\angle -45° \text{ A} \quad (1)$$

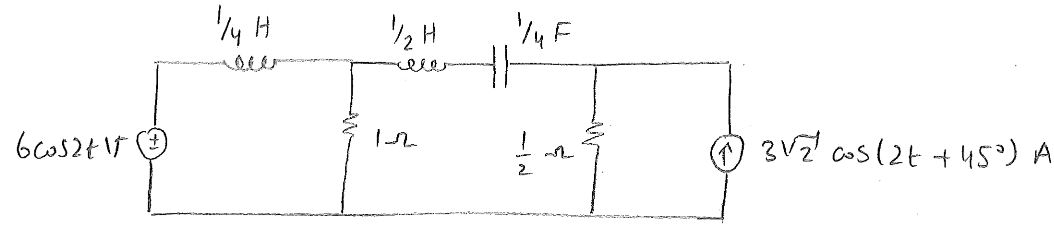
current division $\Rightarrow$ $$I_2 = \frac{j200}{j200 - j600} \cdot I_x = -\frac{1}{2} I_x = -\frac{1}{200}\angle -45° = \frac{1}{200}\angle 135° \text{ A}$$

$$\Rightarrow V_y = -j800\, I_2 = 800\angle -90° \cdot \frac{1}{200}\angle 135° = 4\angle 45° \text{ V} \quad (2)$$

$$(1) \Rightarrow i_x(t) = \mathrm{Re}\{ I_x e^{j\omega t} \} = \boxed{10\cos\left(1000t - \frac{\pi}{4}\right) \text{ mA}}$$

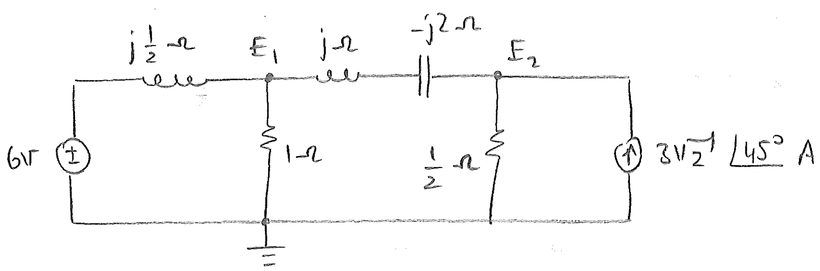
$$(2) \Rightarrow \boxed{v_y(t) = 4\cos\left(1000t + \frac{\pi}{4}\right) \text{ V}}$$

Example [Node analysis]



Find the steady state voltage across the 1Ω resistor.

Sol'n



$[w = 2 \text{rad/sec}]$

Node ①: $\dfrac{E_1 - 6}{j\frac{1}{2}} + \dfrac{E_1}{1} + \dfrac{E_1 - E_2}{-j} = 0 \quad (1)$

Node ②: $\dfrac{E_2 - E_1}{-j} + \dfrac{E_2}{1/2} - 3\sqrt{2}\angle 45° = 0 \quad (2)$

$(1) \Rightarrow \{-j2 + 1 + j\} E_1 - jE_2 = -j12 \Rightarrow (1-j)E_1 - jE_2 = -j12 \Rightarrow E_2 = -(1+j)E_1 + 12 \quad (3)$

$(2) \Rightarrow -jE_1 + (2+j)E_2 = 3 + j3 \quad (4)$

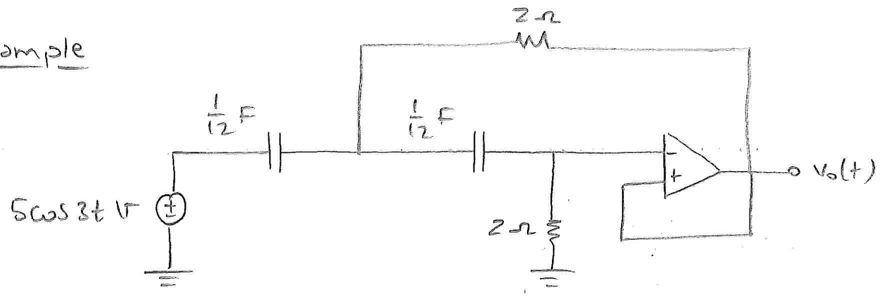
$(3)\&(4) \Rightarrow -jE_1 + (2+j)\{12 - (1+j)E_1\} = 3 + j3$

$\Rightarrow -\{j + (1+j)(2+j)\} E_1 = -12(2+j) + 3 + j3$

$\Rightarrow -(1+j4)E_1 = -21 - j9 \Rightarrow E_1 = \dfrac{21+j9}{1+j4} = \dfrac{57-j75}{17} = \dfrac{3}{17}(19 - j25) \quad (5)$

Finally, (5) $\Rightarrow$ $\boxed{e_1(t) = \dfrac{3}{17}\sqrt{19^2 + 25^2} \cos(2t - \tan^{-1}(25/19)) \text{ V}}$

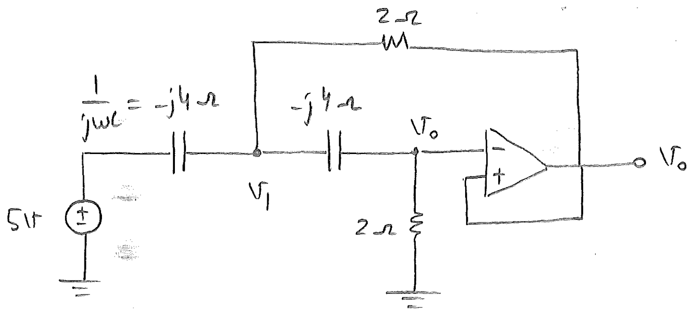
Example



Assume: OPAMP is in linear region & the circuit is in SSS.

Find $v_o(t)$.

Sol'n



$[\omega = 3 \text{rad/sec}]$

$$\frac{V_1 - 5}{-j4} + \frac{V_1 - V_o}{-j4} + \frac{V_1 - V_o}{2} = 0 \quad \Rightarrow \quad \left\{\frac{1}{2} + j\frac{1}{2}\right\} V_1 - \left\{\frac{1}{2} + j\frac{1}{4}\right\} V_o = j\frac{5}{4} \qquad (1)$$

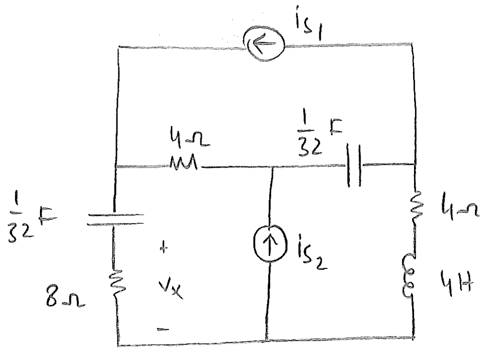
$$\frac{V_o - V_1}{-j4} + \frac{V_o}{2} = 0 \quad \Rightarrow \quad \left\{\frac{1}{2} + j\frac{1}{4}\right\} V_o - j\frac{1}{4} V_1 = 0 \quad \Rightarrow \quad V_1 = (1 - j2) V_o \qquad (2)$$

$$(1)\&(2) \Rightarrow \left[\left(\frac{1}{2} + j\frac{1}{2}\right)(1 - j2) - \left(\frac{1}{2} + j\frac{1}{4}\right)\right] V_o = j\frac{5}{4} \quad \Rightarrow \quad \left(1 - j\frac{3}{4}\right) V_o = j\frac{5}{4}$$

$$\Rightarrow \frac{5}{4}\angle -\tan^{-1} 3/4 \; V_o = \frac{5}{4}\angle 90^\circ \quad \Rightarrow \quad V_o = 1 \angle 90^\circ + \tan^{-1}(3/4) \text{ V} \qquad (3)$$

$$(3) \Rightarrow \boxed{v_o(t) = \cos\left(3t + 90^\circ + \tan^{-1}\frac{3}{4}\right) \text{ V}}$$

Example [Mesh Analysis]



Given :

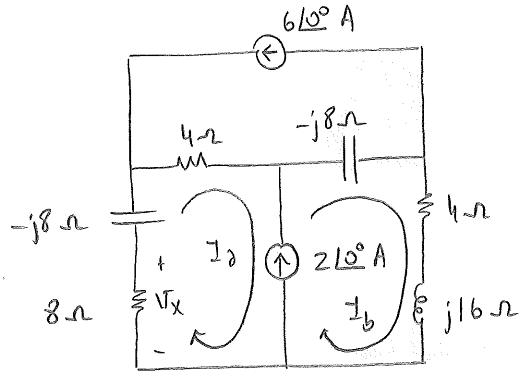
→ $i_{s_1}(t) = 6\cos 4t$ A

→ $i_{s_2}(t) = 2\cos 4t$ A

→ circuit is in SSS

Find $v_x(t)$.

Sol'n [$\omega = 4$ rad/sec]



Supermesh :

$(8-j8)I_a + 4(I_a+6) - j8(I_b+6) + (4+j16)I_b = 0$ (1)

constraint : $I_b - I_a = 2 \Rightarrow I_b = I_a + 2$ (2)

(1)&(2) $\Rightarrow (8-j8+4)I_a + (-j8+4+j16)(I_a+2) = -24+j48$

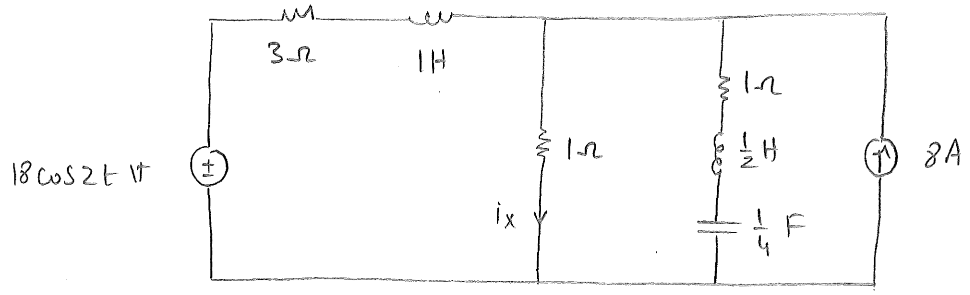
$\Rightarrow (12-j8)I_a + (4+j8)I_a = -24+j48-2(4+j8)$

$\Rightarrow 16I_a = -32+j32 \Rightarrow I_a = -2+j2$ A

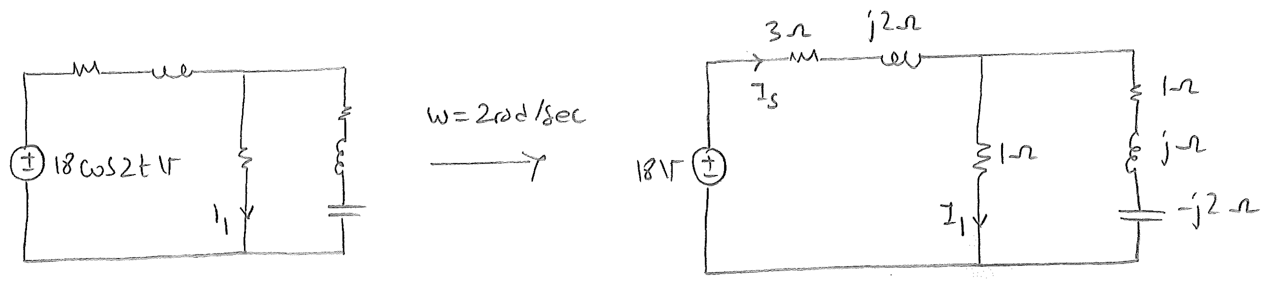
$\Rightarrow V_x = -8I_a = 16-j16 = 16\sqrt{2}\angle -45°$ V

$\Rightarrow \boxed{v_x(t) = 16\sqrt{2}\cos\left(4t - \frac{\pi}{4}\right) \text{ V}}$

Example [Superposition] Find $i_x(t)$ in the steady state.



Step 1 kill DC source.

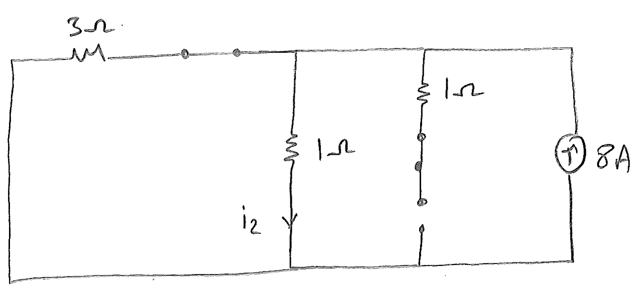


$$I_s = \frac{18}{(3+j2) + (1 \,//\, (1-j))} = \frac{18}{3.6+j1.8} = 4-j2$$

$$\Rightarrow I_1 = \frac{1-j}{1+(1-j)} \cdot I_s = \frac{3-j}{5}(4-j2) = 2-j2 = 2\sqrt{2}\angle{-45^\circ}\ A$$

$$\Rightarrow i_1(t) = 2\sqrt{2}\cos(2t-45^\circ)\ A$$

Step 2 kill AC source.

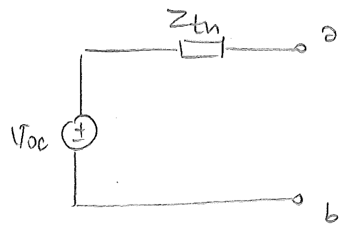


$i_2(t) = 6A$

Step 3 superpose. $i_x(t) = i_1(t) + i'_2(t) = \boxed{6 + 2\sqrt{2}\cos(2t-45^\circ)\ A}$

Thevenin / Norton Equivalent Circuits

Just like in the LTI resistive case.



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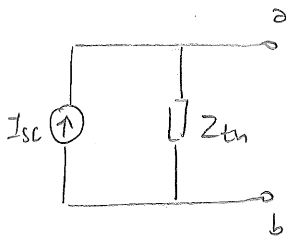
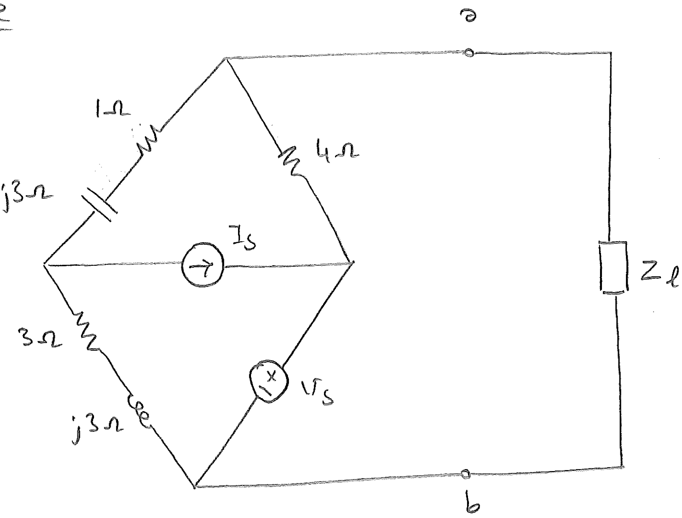


$$Z_{th} = \frac{V_{oc}}{I_{sc}}$$

Thevenin Equivalent in phasor domain

Norton Equivalent

Example

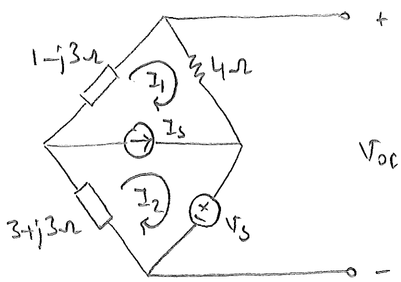


$I_s = \sqrt{2}\,\angle{-45°}$ A

$V_s = 8\,\angle 0°$ V

Find the Thevenin equiv. circuit seen by the load Z_ℓ.

Sol'n First, find V_{oc}.



Supermesh: $(1-j3)I_1 + 4I_1 + V_s + (3+j3)I_2 = 0$

$\Rightarrow (5-j3)I_1 + (3+j3)I_2 = -8 \qquad (1)$

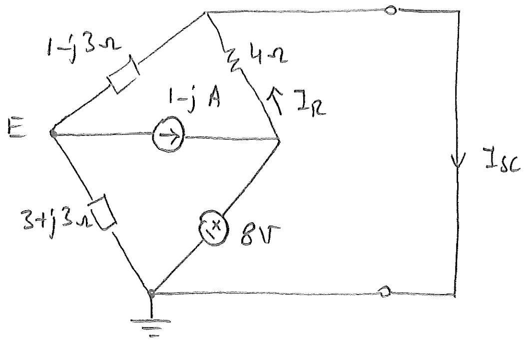
Constraint: $I_2 - I_1 = I_s \Rightarrow I_2 = I_1 + (1-j) \qquad (2)$

$(1)\&(2) \Rightarrow (5-j3)I_1 + (3+j3)\{I_1 + (1-j)\} = -8 \Rightarrow 8I_1 = -8-(3+j3)(1-j) = -14$

$\Rightarrow I_1 = -\frac{7}{4}$ A

$\Rightarrow V_{oc} = 4I_1 + V_s = 4\left(-\frac{7}{4}\right) + 8 = \boxed{1V}$

Now, find I_{sc}.



$$\frac{E}{3+j3} + (1-j) + \frac{E}{1-j3} = 0$$

$$\Rightarrow \left(\frac{1}{3+j3} + \frac{1}{1-j3}\right)E = -1+j$$

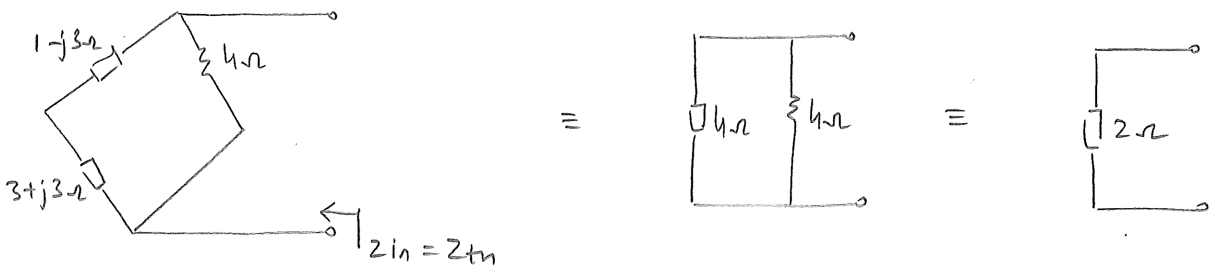
$$\Rightarrow \frac{4+j2}{15} E = -1+j \quad \Rightarrow E = -\frac{3}{2} + j\frac{9}{2} \ V$$

$$\& \quad I_R = \frac{8}{4} = 2A$$

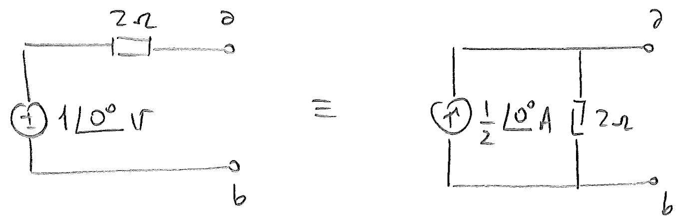
$$\Rightarrow I_{sc} = \frac{E}{1-j3} + I_R = -\frac{3}{2}(1-j3)\cdot\frac{1}{1-j3} + 2 = \boxed{\frac{1}{2} A}$$

Hence $Z_{th} = \frac{V_{oc}}{I_{sc}} = \frac{1}{1/2} = \boxed{2\Omega}$

Alternatively, we can compute Z_{th} by killing the ind. sources:

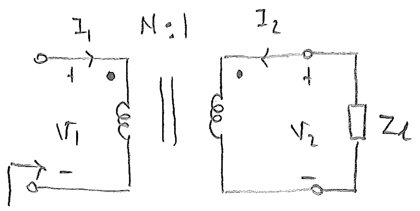


Finally, Z_ℓ sees:



Thev. Equiv. Norton Equiv.

Example



$Z_{in} = ?$

$$Z_{in} = \frac{V_1}{I_1} = \frac{NV_2}{-I_2/N} = \frac{N(-I_2 Z_\ell)}{-I_2/N} = N^2 Z_\ell$$

$$\Rightarrow \boxed{Z_{in} = N^2 Z_\ell}$$