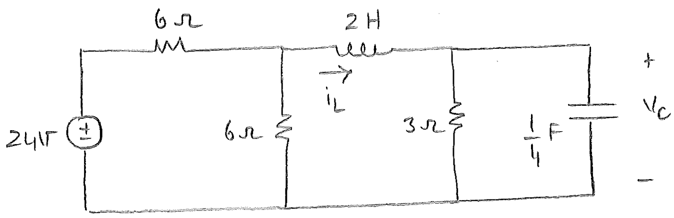
**Definition** A circuit is said to be in DC steady state if all its branch currents and branch voltages are constant.

**Example** The following circuit has reached DC steady state. Find $v_c$ and $i_L$.

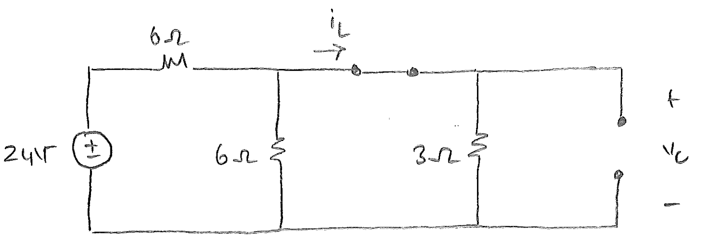



**Sol'n** Observe that in DC steady state

$$i_L(t) = \text{constant} \Rightarrow v_L(t) = L \cancel{\frac{di_L(t)}{dt}}^{\,0} = 0$$

$$v_c(t) = \text{constant} \Rightarrow i_c(t) = C \cancel{\frac{dv_c(t)}{dt}}^{\,0} = 0$$

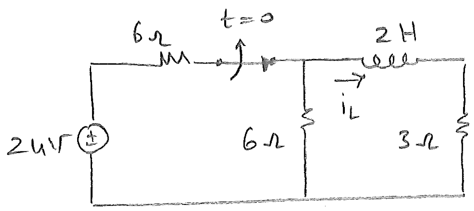
Therefore we can consider the following circuit.




$\Rightarrow i_L(t) = 2A$ & $v_c(t) = 6V$ in DC steady state.

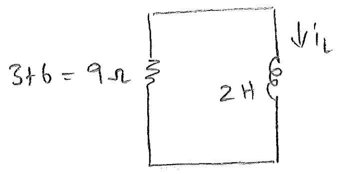
**Remark** In DC steady state inductors behave as short circuit and capacitors as open circuit.

**Example** The circuit is initially (switch closed) in DC steady state. Find $i_L(t)$ for $t>0$ (switch open).




**Sol'n** $i_L(0) = 2A$ (from previous example)

For $t>0$ we have



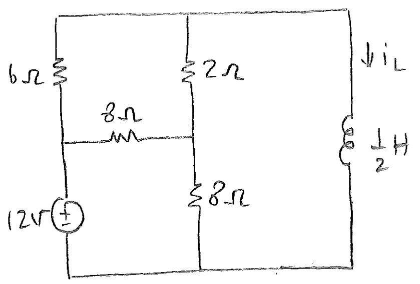

$i_L(0) = 2A$

$$i_L(t) = i_L(0)e^{-t/\tau}$$

time constant $\tau = \frac{L}{R} = \frac{2}{9}$ s

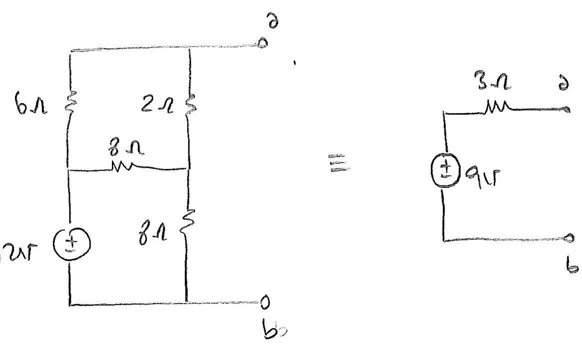
$$\Rightarrow \boxed{i_L(t) = 2e^{-9t/2} \text{ A}}$$

**Example** Find $i_L(t)$ for $t \geq 0$.

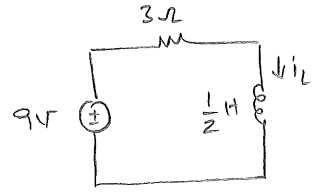



$i_L(0) = -2A$

**Step 1** Find Thevenin / Norton equiv.



Step 2 Consider the equiv. circuit




$i_L(0) = -2A$

$$9 = 3i_L + \frac{1}{2}Di_L \Rightarrow Di_L + 6i_L = 18 \qquad (1)$$

$\Rightarrow$ nat. freq. $s = -6$

$\Rightarrow$ hmg. sol'n $i_h(t) = Ke^{-6t}$

$i_p(t) = ?$ $i_p(t) =$ constant (because the right-hand side of (1) is constant)

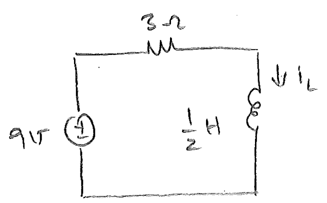
$$(1) \Rightarrow \cancel{Di_p}^{0} + 6i_p = 18 \Rightarrow i_p(t) = 3$$

$$\Rightarrow i_L(t) = i_p(t) + i_h(t) = 3 + Ke^{-6t}$$

$K = ?$ $i_L(0) = -2 \Rightarrow 3 + Ke^{-6t}\Big|_{t=0} = -2 \Rightarrow K = -5$

$$\Rightarrow \boxed{i_L(t) = 3 - 5e^{-6t}\ A}$$

Step 2 (Alternative)




$i_L(0) = -2$ Amps

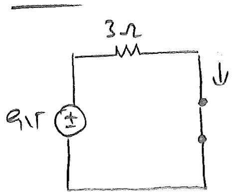
$i_L(t) = A + Be^{-t/\tau}$ $(A, B, \tau = ?)$

$$\tau = \frac{L}{R} = \frac{1/2}{3} = \frac{1}{6}\ s$$

$$\Rightarrow i_L(t) = A + Be^{-6t}$$

Note that as $t \to \infty$ $i_L(t) \to A$ $\Rightarrow$ $i_L(t)$ becomes constant $\Rightarrow$ circuit reaches DC steady state

$t = \infty$

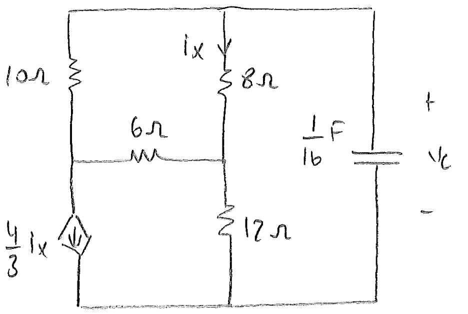



$i_L(\infty) = 3$ Amps $\Rightarrow A = 3 \Rightarrow i_L(t) = 3 + Be^{-6t}$

Finally find B by using init. cond. constraint

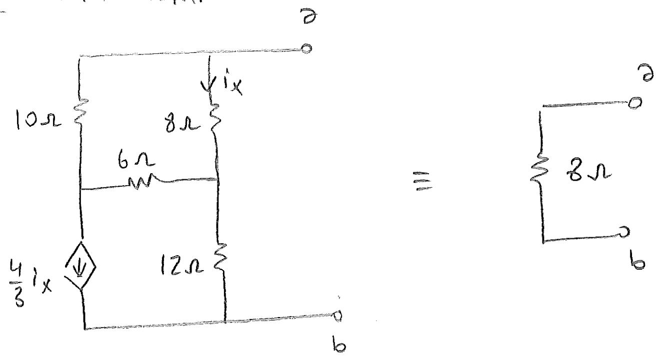
$$i_L(0) = -2 \text{ Amps} \Rightarrow 3 + Be^{-6t}\Big|_{t=0} = -2 \Rightarrow B = -5 \Rightarrow \boxed{i_L(t) = 3 - 5e^{-6t}\ \text{Amps}}$$

Example Find $i_x(t)$ for $t \geq 0$.

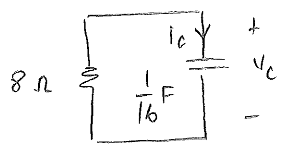



$v_c(0) = 6V$

Step1 Find $R_{th}$.




Step2 Solve the RC circuit

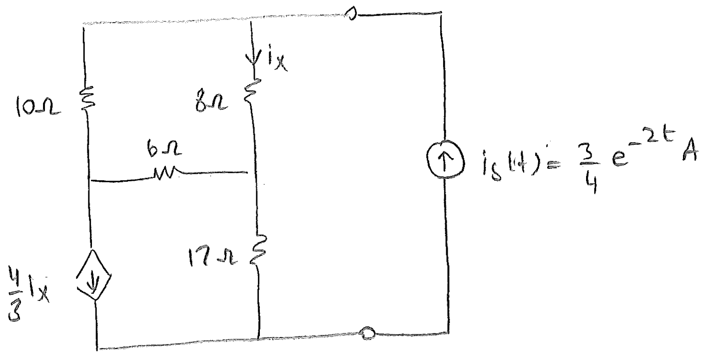



$v_c(0) = 6V$

$$\Rightarrow \quad v_c(t) = v_c(0)\, e^{-t/RC} = 6e^{-2t} \text{ V}$$

$$\Rightarrow \quad i_c(t) = C\, D v_c(t) = \frac{1}{16}\{-12e^{-2t}\} = -\frac{3}{4}e^{-2t} \text{ A}$$

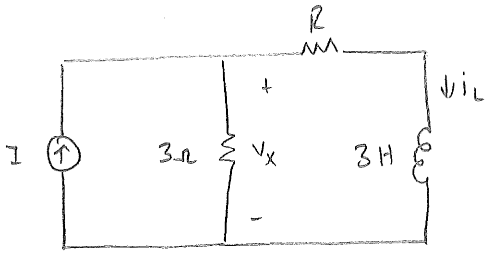
Step3 Substitute capacitor with a ind. source and solve for $i_x$.




$$\Rightarrow \quad i_x = \frac{1}{2} i_s$$

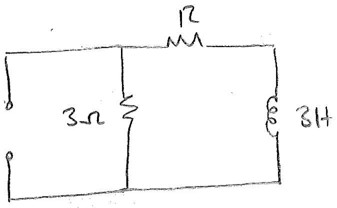
$$\Rightarrow \quad \boxed{i_x(t) = \frac{3}{8} e^{-2t} \text{ A}}$$

**Example** Consider the below circuit where $I$ is constant. Find $i_L(0)$, $I$, $R$.

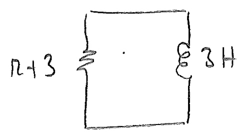



$$V_x(t) = 12 - 9e^{-3t}\ V \qquad (1)$$

**Sol'n** Note that (1) $\Rightarrow$ $\tau = \frac{1}{3}$ s. Since the time constant $\tau$ does not depend on the input consider the input-free circuit. (I.e. kill the current source.)




=



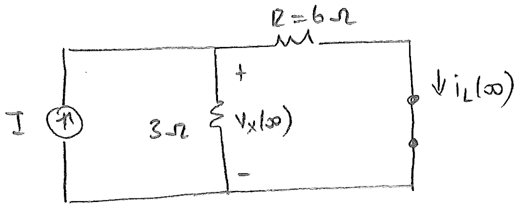

$\Rightarrow \tau = \frac{3}{R+3} = \frac{1}{3}$

$\Downarrow$

This circuit & the original circuit have the same $\tau$

$\Rightarrow \boxed{R = 6\ \Omega}$

**$t=\infty$** At $t=\infty$ circuit reaches DC steady state. (I.e. inductor becomes short circuit.)

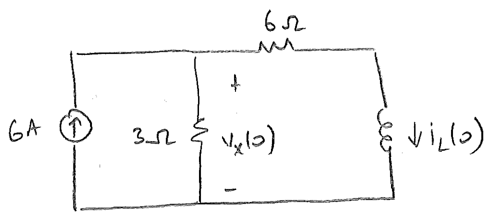



(1) $\Rightarrow$ $V_x(\infty) = 12\ V$

$V_x(\infty) = I \times (6 \parallel 3) = 2I$.

$\Rightarrow \boxed{I = 6A}$

**$t=0$**




KCL $\Rightarrow$ $6 = \frac{V_x(0)}{3} + i_L(0)$

$\Rightarrow \boxed{i_L(0) = 5A}$

(1) $\Rightarrow$ $V_x(0) = 3\ V$