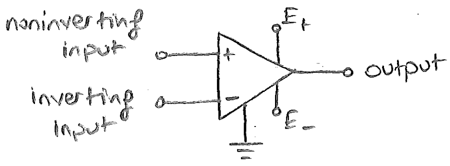
#1 2 Kasım 2023

Ch. III

Operational Amplifiers

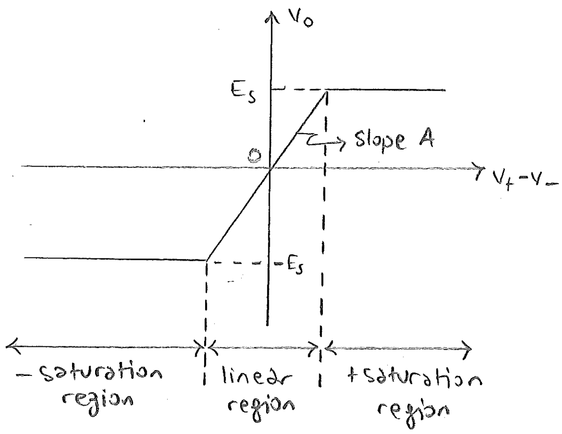
Six (or more) terminal device



E_+ & E_- terminals are connected to DC voltage supplies. The values $E_+ > E_-$ determine the upper & lower limits of the output voltage v_o.

That is, $E_- \leq v_o \leq E_+$

Transfer Characteristics (let $E_+ = E_s$ & $E_- = -E_s$)



A : open-loop voltage gain

Typically $A \cong 200{,}000$

v_o : voltage of the output terminal
v_+ : voltage of the ⊕ terminal
v_- : voltage of the ⊖ terminal

w.r.t. some common ground

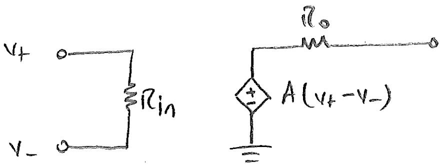
OP-AMP has three operating modes:

1) linear mode, $v_o = A(v_+ - v_-)$ and $-E_s < v_o < E_s$

2) +sat mode, $v_o = E_s$ and $A(v_+ - v_-) > E_s$

3) –sat mode, $v_o = -E_s$ and $A(v_+ - v_-) < -E_s$

Dependent Source model of an OPAMP operating in linear region:



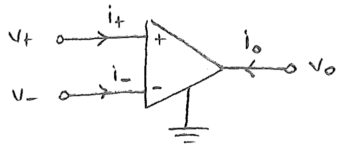
R_{in} : input resistance, $\sim 10^6 - 10^{12}\ \Omega$

R_o : output resistance, $\sim 10 - 100\ \Omega$

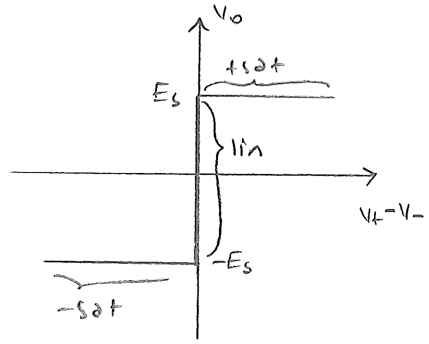
A : voltage gain, $\sim 10^5 - 10^8$ V/V

<u>Remark</u> For many applications it is reasonable to work with the ideal OP-AMP model. ($R_{in} = \infty$, $R_o = 0$, $A = \infty$)

Infinite-gain ideal OP-AMP



linear region	+sat region	-sat region
$V_+ = V_-$	$V_+ > V_-$	$V_+ < V_-$
$-E_s \le V_o \le E_s$	$V_o = E_s$	$V_o = -E_s$

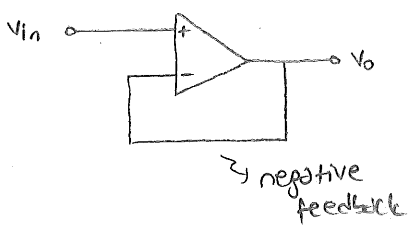


$i_+ = i_- = 0$ (all regions)

Note : $i_o \neq 0$!

Some useful OP-AMP circuits:

Voltage follower (Buffer)



$V_+ = V_{in}$, $V_- = V_o$

linear $V_+ = V_-$ & $-E_s \le V_o \le E_s$

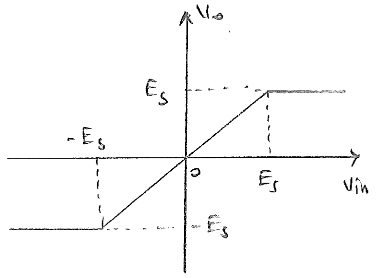
$\Rightarrow$ $V_o = V_{in}$ & $-E_s \le V_{in} \le E_s$

+sat $V_+ > V_-$ & $V_o = E_s$

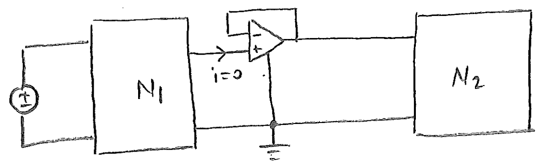
$\Rightarrow$ $V_{in} > E_s$

-sat $V_+ < V_-$ & $V_o = -E_s$

$\Rightarrow$ $V_{in} < -E_s$



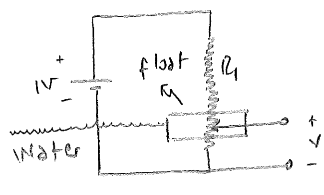
Remark Buffers are widely used to isolate 2 two-ports



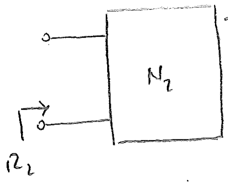
Here, the buffer prevents N_2 from "loading down" N_1.

Ex :

N_1 :



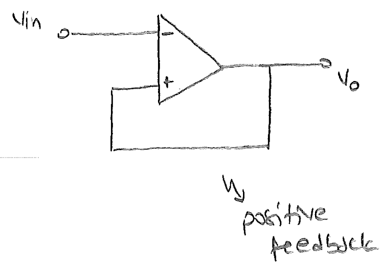
voltage proportional to water level

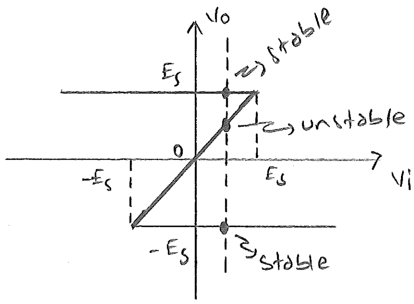


circuitry that controls the water pump to regulate the water level

where $R_2 \ll R_1$

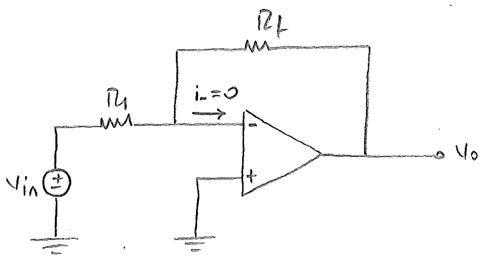
How about ?





[i.e. in practice the OP-AMP will quickly jump to either to +sat or to -sat region under any disturbance.]

Inverting amplifier



KCL : $\dfrac{V_- - V_{in}}{R_1} + \dfrac{V_- - V_o}{R_f} = 0$

$$\Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_f}\right) V_- = \frac{1}{R_1} V_{in} + \frac{1}{R_f} V_o$$

$$\Rightarrow V_- = \left(\frac{1}{R_1} + \frac{1}{R_f}\right)^{-1} \left\{ \frac{1}{R_1} V_{in} + \frac{1}{R_f} V_o \right\} \quad (1)$$

$V_+ = 0$ (2). Note that (1) & (2) are valid at all regions.

linear $\quad V_- = V_+$ & $-E_s \le V_o \le E_s$

$$\left(\frac{1}{R_1} + \frac{1}{R_f}\right)^{-1} \left\{ \frac{1}{R_1} V_{in} + \frac{1}{R_f} V_o \right\} = 0 \quad \Rightarrow \quad \boxed{V_o = -\frac{R_f}{R_1} V_{in}}$$

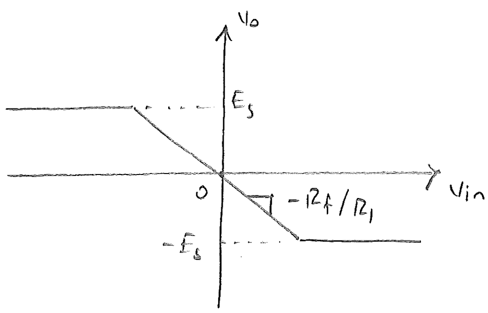
& $\quad -E_s \le -\dfrac{R_f}{R_1} V_{in} \le E_s \quad \Rightarrow \quad \boxed{-\dfrac{R_1}{R_f} E_s \le V_{in} \le \dfrac{R_1}{R_f} E_s}$

+sat $\quad V_- < V_+$ & $\boxed{V_o = E_s}$

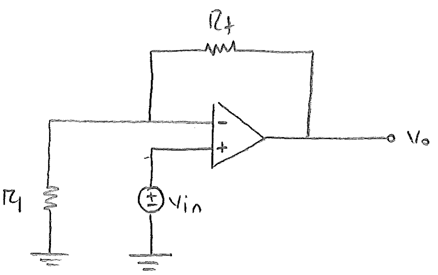
$$\left(\frac{1}{R_1} + \frac{1}{R_f}\right)^{-1} \left\{ \frac{1}{R_1} V_{in} + \frac{1}{R_f} E_s \right\} < 0 \quad \Rightarrow \quad \boxed{V_{in} < -\frac{R_1}{R_f} E_s}$$

−sat $\quad V_- > V_+$ & $\boxed{V_o = -E_s} \quad \Rightarrow \quad \boxed{V_{in} > \dfrac{R_1}{R_f} E_s}$

Hence,



Noninverting amplifier



KCL : $\frac{v_-}{R_1} + \frac{v_- - v_o}{R_f} = 0 \Rightarrow v_- = \left(1+\frac{R_f}{R_1}\right)^{-1} v_o$ (1)

& $v_+ = v_{in}$ (2)

linear $v_- = v_+$ & $-E_s \leq v_o \leq E_s$

$$\Rightarrow \left(1+\frac{R_f}{R_1}\right)^{-1} v_o = v_{in} \Rightarrow \boxed{v_o = \left(1+\frac{R_f}{R_1}\right) v_{in}}$$

$$\& \quad -E_s \leq \left(1+\frac{R_f}{R_1}\right) v_{in} \leq E_s \Rightarrow \boxed{-\left(1+\frac{R_f}{R_1}\right)^{-1} E_s \leq v_{in} \leq \left(1+\frac{R_f}{R_1}\right) E_s}$$

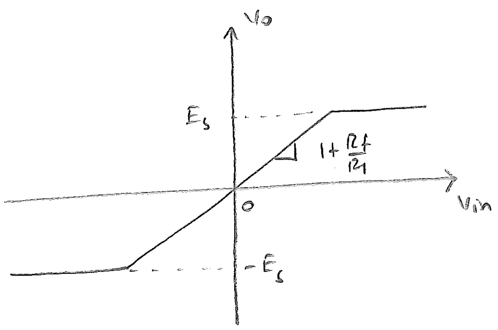
+sat $v_- < v_+$ & $\boxed{v_o = E_s}$

$$\boxed{\left(1+\frac{R_f}{R_1}\right)^{-1} E_s < v_{in}}$$

−sat $v_- > v_+$ & $\boxed{v_o = -E_s}$

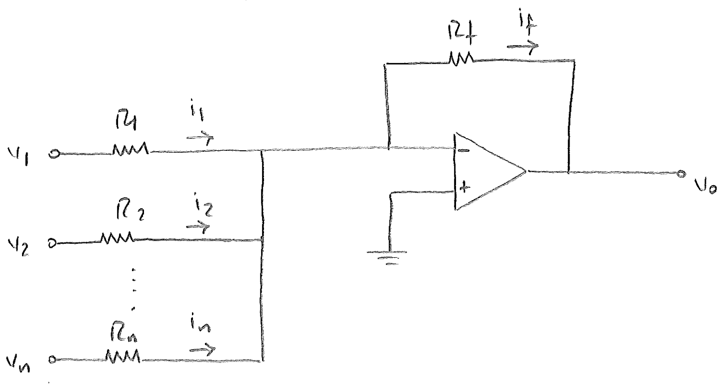
$$\boxed{-\left(1+\frac{R_f}{R_1}\right)^{-1} E_s > v_{in}}$$

Hence,



Remark Note that the voltage follower is a special case of the noninverting amplifier with $R_f = 0$ & $R_1 = \infty$.

Summing amplifier



In linear region $v_- = v_+ = 0$

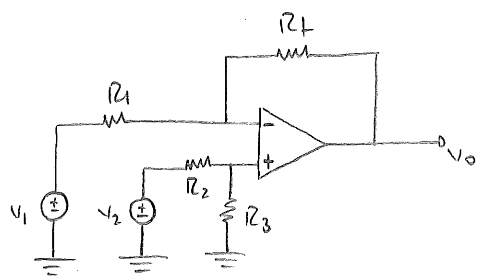
$$\Rightarrow i_k = \frac{v_k - v_-}{R_k} = \frac{v_k}{R_k} \quad \& \quad i_f = \frac{v_- - v_o}{R_f} = -\frac{v_o}{R_f}$$

KCL $\Rightarrow$ $i_1 + i_2 + \cdots + i_n = i_f$

$$\Rightarrow \frac{v_1}{R_1} + \frac{v_2}{R_2} + \cdots + \frac{v_n}{R_n} = -\frac{v_o}{R_f}$$

$$\Rightarrow \boxed{v_o = -\left\{ \frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \cdots + \frac{R_f}{R_1} v_n \right\}} \quad \text{(in linear region)}$$

Difference amplifier



$$v_+ = \frac{R_3}{R_2 + R_3} v_2 = \left[1 + \frac{R_2}{R_3} \right]^{-1} v_2 \quad (1)$$

$$\frac{v_- - v_1}{R_1} + \frac{v_- - v_o}{R_f} = 0 \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_f} \right) v_- = \frac{v_1}{R_1} + \frac{v_o}{R_f}$$

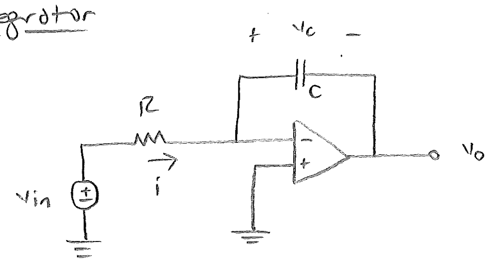
$$\Rightarrow v_o = \frac{R_f}{R_1} \left\{ \left[1 + \frac{R_1}{R_f} \right] v_- - v_1 \right\} \quad (2)$$

In linear region $v_+ = v_-$

$$(1) \& (2) \Rightarrow v_o = \frac{R_f}{R_1} \left\{ \left[1 + \frac{R_1}{R_f} \right] \left[1 + \frac{R_2}{R_3} \right]^{-1} v_2 - v_1 \right\}$$

Then choosing $\frac{R_1}{R_f} = \frac{R_2}{R_3}$ yields $\boxed{v_o = \frac{R_f}{R_1}(v_2 - v_1)}$ (in lin. region)

Integrator



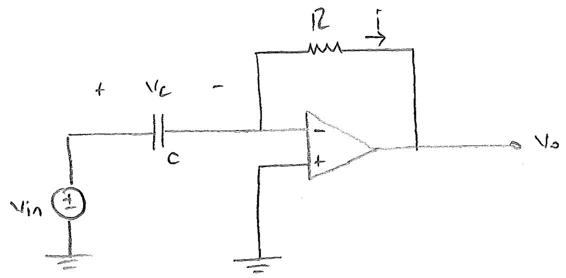
In linear region $V_- = V_+ = 0$

$\Rightarrow \quad i = \frac{V_{in}}{R} \quad (1) \quad$ & $\quad V_C = -V_o \quad (2)$

Also, $V_C(t) = V_C(0) + \frac{1}{C}\int_0^t i(\tau)d\tau \quad (3)$

$(1),(2),(3) \Rightarrow \quad \boxed{V_o(t) = V_o(0) - \frac{1}{RC}\int_0^t V_{in}(\tau)d\tau}$ (in lin. region)

Differentiator

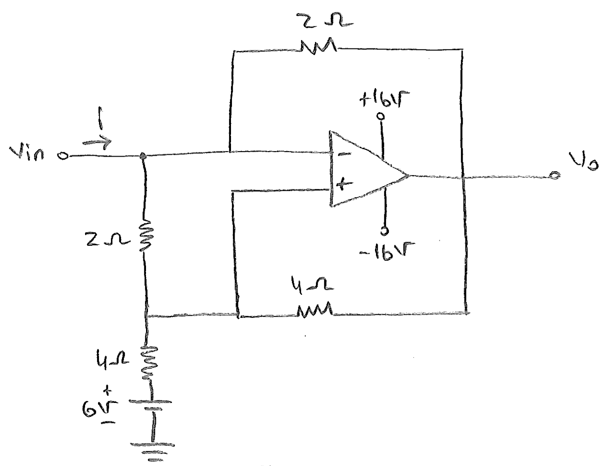


In linear region $V_- = V_+ = 0$

$$\left.\begin{array}{l} V_{in} = V_C \\ i = C\frac{dV_C}{dt} = C\frac{dV_{in}}{dt} \\ V_o = -Ri \end{array}\right\} \quad \boxed{V_o(t) = -RC\frac{dV_{in}}{dt}}$$

(in lin. region)

Example: obtain the input ($V_{in}-i$) & transfer ($V_{in}-V_o$) characteristics.



$V_- = V_{in}$ (1)

$V_+ = ?$

$$\frac{V_+ - V_{in}}{2} + \frac{V_+ - 6}{4} + \frac{V_+ - V_o}{4} = 0$$

$$\Rightarrow \left\{\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right\} V_+ = \frac{1}{2}V_{in} + \frac{1}{4}V_o + \frac{3}{2}$$

$$\Rightarrow V_+ = \frac{1}{2}V_{in} + \frac{1}{4}V_o + \frac{3}{2} \quad (2)$$

$i = ?$

$$i = \frac{V_{in} - V_+}{2} + \frac{V_{in} - V_o}{2}$$

$$= V_{in} - \frac{1}{2}V_o - \frac{1}{2}\left\{\frac{1}{2}V_{in} + \frac{1}{4}V_o + \frac{3}{2}\right\}$$

$$\Rightarrow i = \frac{3}{4}V_{in} - \frac{5}{8}V_o - \frac{3}{4} \quad (3)$$

Eq. (1), (2), (3) are valid in all regions!

Linear region: $V_- = V_+$ & $-16 \le V_o \le 16$

$$V_- = V_+ \Rightarrow V_{in} = \frac{1}{2}V_{in} + \frac{1}{4}V_o + \frac{3}{2} \Rightarrow \boxed{V_o = 2V_{in} - 6}$$

$$-16 \le V_o \le 16 \Rightarrow -16 \le 2V_{in} - 6 \le 16 \Rightarrow \boxed{-5 \le V_{in} \le 11}$$

$$(3) \Rightarrow i = \frac{3}{4}V_{in} - \frac{5}{8}(2V_{in} - 6) - \frac{3}{4} \Rightarrow \boxed{i = -\frac{1}{2}V_{in} + 3}$$

+sat $\quad v_- < v_+ \quad \& \quad \boxed{v_o = 16V}$

$$v_- < v_+ \Rightarrow v_{in} < \frac{1}{2}v_{in} + \frac{1}{4}(16) + \frac{3}{2} \Rightarrow \boxed{v_{in} < 11}$$

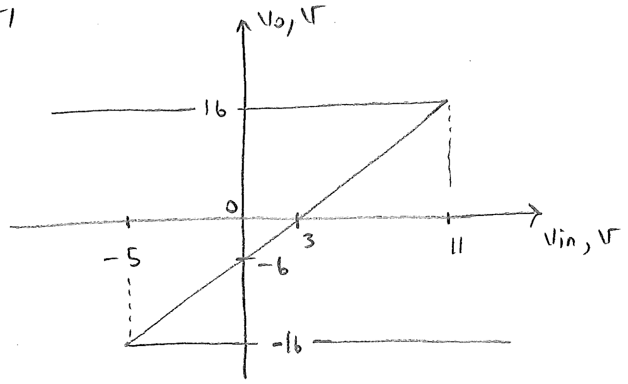
$$(3) \Rightarrow i = \frac{3}{4}v_{in} - \frac{5}{8}(16) - \frac{3}{4} \Rightarrow \boxed{i = \frac{3}{4}v_{in} - \frac{43}{4}}$$

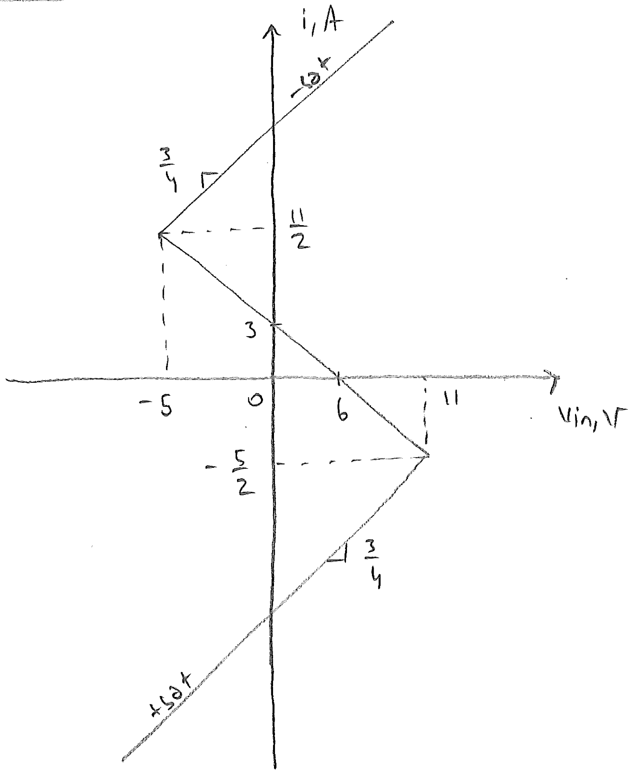
−sat $\quad v_- > v_+ \quad \& \quad \boxed{v_o = -16V}$

$$v_- > v_+ \Rightarrow v_{in} > \frac{1}{2}v_{in} + \frac{1}{4}(-16) + \frac{3}{2} \Rightarrow \boxed{v_{in} > -5}$$

$$(3) \Rightarrow i = \frac{3}{4}v_{in} - \frac{5}{8}(-16) - \frac{3}{4} \Rightarrow \boxed{i = \frac{3}{4}v_{in} + \frac{37}{4}}$$

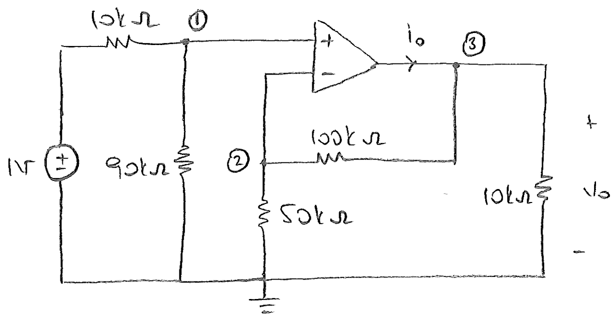
Hence,





#9 2 Kasım 2023

Example [A&S] Suppose the OP-AMP is in linear region. Find v_o and i_o.



Sol'n

Node 1 : $\frac{V_+ - 1}{10k} + \frac{V_+}{90k} = 0 \Rightarrow V_+ = 0.9V$

Node 2 : $\frac{V_-}{50k} + \frac{V_- - V_o}{100k} = 0$

$V_- = V_+$

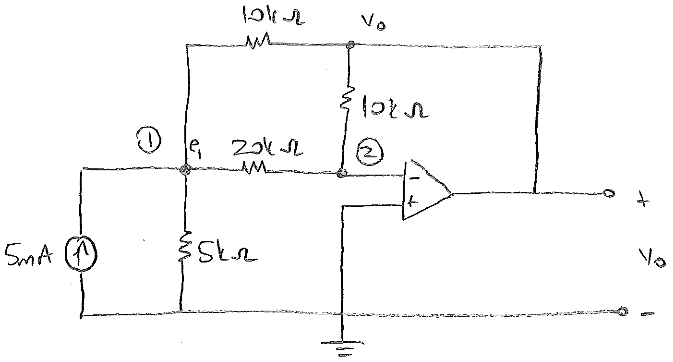
$\Rightarrow \frac{0.9}{50k} + \frac{0.9 - V_o}{100k} = 0$

$\Rightarrow \boxed{V_o = 2.7V}$

Node 3 : $i_o = \frac{V_o - V_-}{100k} + \frac{V_o}{10k} = \frac{2.7 - 0.9}{100k} + \frac{2.7}{10k} = \frac{288}{10^6}$

$\Rightarrow \boxed{i_o = 288\mu A}$

Example [A&S] Suppose the op-AMP is in linear region. Find v_o.



$v_+ = 0$

$\Rightarrow v_- = 0$

Node ② : $\frac{v_- - e_1}{20k} + \frac{v_- - v_o}{10k} = 0 \Rightarrow -\frac{e_1}{20k} - \frac{v_o}{10k} = 0$

$\Rightarrow e_1 = -2v_o \quad (1)$

Node ① : $-5m + \frac{e_1}{5k} + \frac{e_1 - v_-}{20k} + \frac{e_1 - v_o}{10k} = 0$

$v_- = 0$
$e_1 = -2v_o$

$\Rightarrow -5 + \frac{-2v_o}{5} + \frac{-2v_o}{20} + \frac{-3v_o}{10} = 0$

$\Rightarrow -5 = (0.4 + 0.1 + 0.3)v_o \Rightarrow \boxed{v_o = -6.25V}$