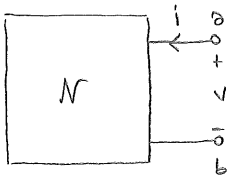
#1 28 Ekim 2023

Thevenin & Norton Equivalent Circuits

Recall: two one-ports are equivalent if they have identical i-v characteristics.

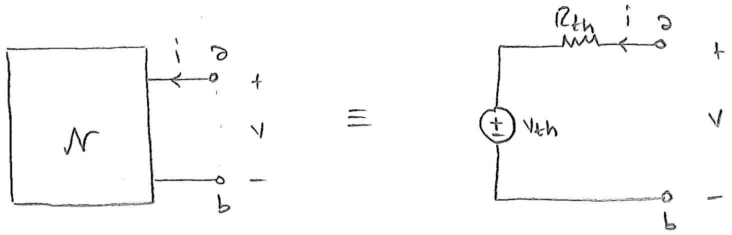
Thevenin's Theorem Consider the below one-port, where $\mathcal{N}$ is an LTI resistive circuit satisfying:



(A1) Any dep. source in $\mathcal{N}$ has its control var. also in $\mathcal{N}$.

(A2) one-port is both voltage- and current-controlled.

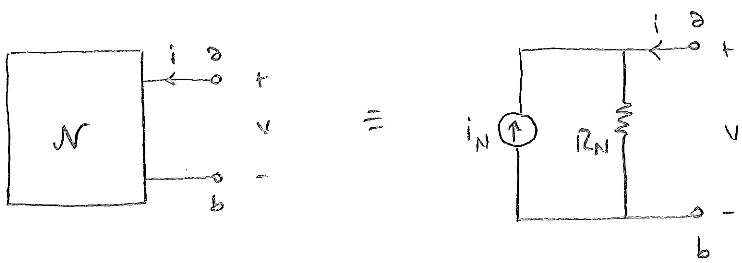
Then we can find v_{th}, R_{th} and establish the below equivalence



i-v characteristics:

$$\boxed{v = R_{th} i + v_{th}} \quad (1)$$

Norton's Theorem Under the same conditions (A1) & (A2) we can find i_N, R_N and the below equivalence holds



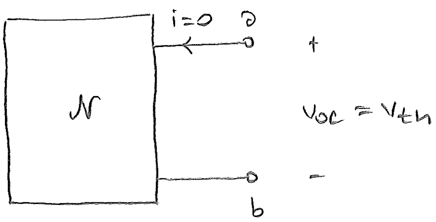
i-v characteristics:

$$\boxed{v = R_N i + R_N i_N} \quad (2)$$

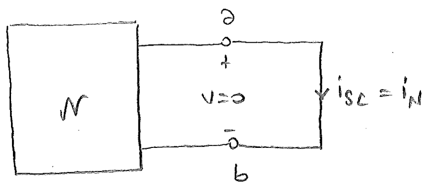
Remark Since (1) ≡ (2) we have to have $R_{th} = R_N$ and $v_{th} = R_{th} i_N$.

Computing v_{th}, i_N, R_{th} (for a given one-port $\mathcal{N}$)

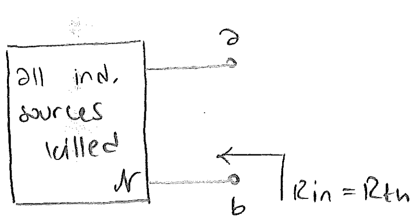
1) Open circuit voltage equals v_{th}:



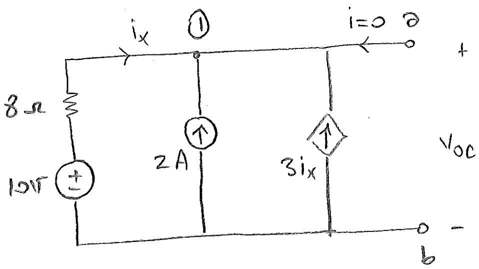
2) Short circuit current equals i_N :



3) Once v_{th} and i_N are known we have $R_{th} = v_{th}/i_N$. Another way to obtain R_{th} is to kill all independent sources within N and compute the input resistance:



Example Find the Thevenin and Norton equivalents of below one-port.



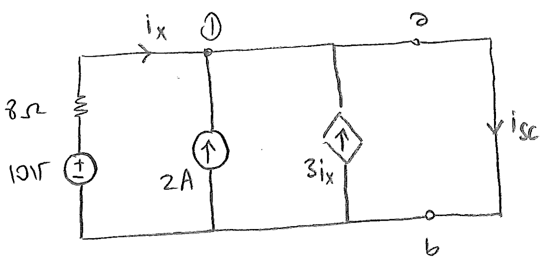
Step 1 Find the open-circuit voltage v_{oc}.

KCL at ①: $i_x + 2 + 3i_x = 0 \Rightarrow i_x = -\frac{1}{2}$ A

$v_{oc} = -8i_x + 10 = 14V$

$\Rightarrow \boxed{v_{th} = 14V}$

Step 2 Find the short-circuit current i_{sc}.



KVL at outer loop : $-10 + 8i_x = 0 \Rightarrow i_x = \frac{5}{4}$ A

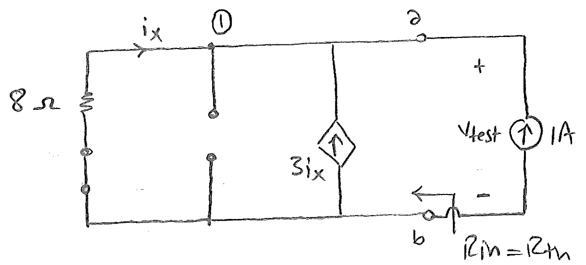
KCL at ① : $i_x + 2 + 3i_x - i_{sc} = 0$

$\Rightarrow i_{sc} = 2 + 4i_x = 7A$

$\Rightarrow \boxed{i_N = 7A}$

Step3 Find R_{th}. $R_{th} = \frac{V_{th}}{i_N} = \frac{14}{7} \Rightarrow \boxed{R_{th} = 2\Omega}$

Alternative way to compute R_{th}: kill all ind. sources.



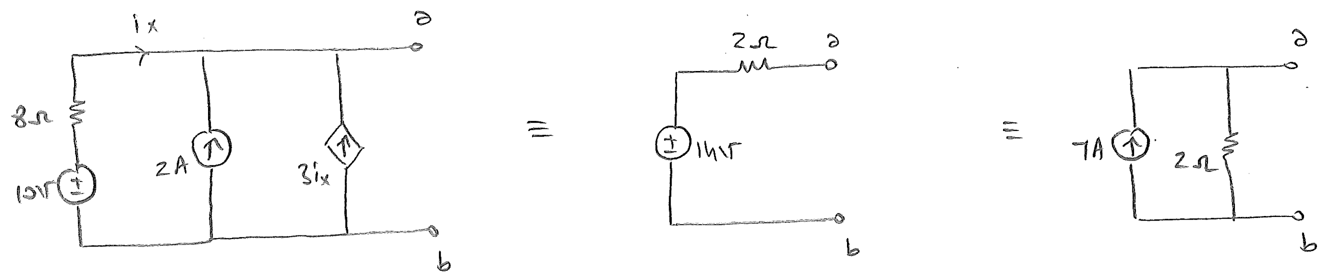
KCL at ①: $i_x + 3i_x + 1 = 0$

$\Rightarrow i_x = -\frac{1}{4} A$

$\Rightarrow V_{test} = -8i_x = 2V$

$\Rightarrow R_{in} = \frac{V_{test}}{1} = 2\Omega \Rightarrow R_{th} = 2\Omega$

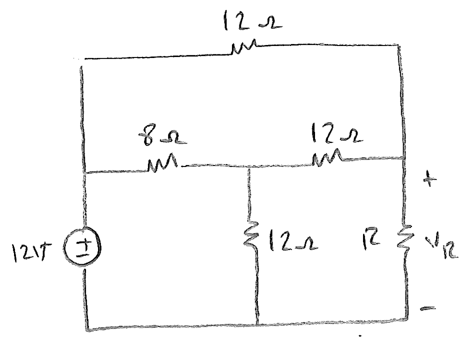
Therefore



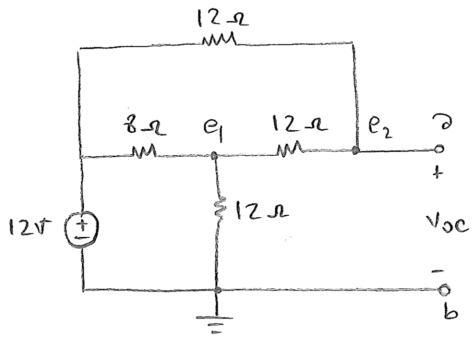
Thevenin Equiv.   Norton Equiv.

Example Find the Thevenin equiv. circuit as viewed by the resistor R.

What should R be so that $V_R = 8V$?



Sol'n First. Remove R and find V_{oc}.



Node ①: $\frac{e_1-12}{8}+\frac{e_1}{12}+\frac{e_1-e_2}{12}=0$

$\Rightarrow \frac{7}{24}e_1-\frac{1}{12}e_2=\frac{3}{2}$ (1)

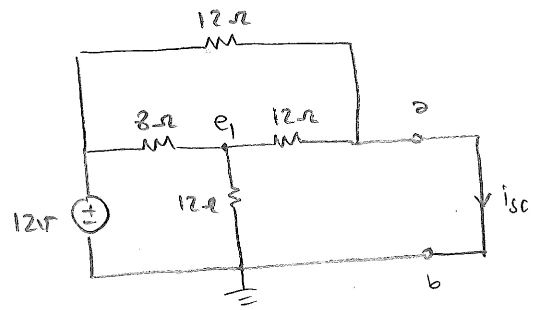
Node ②: $\frac{e_2-e_1}{12}+\frac{e_2-12}{12}=0$

$\Rightarrow -\frac{1}{12}e_1+\frac{1}{6}e_2=1$ (2)

(1) & (2) $\Rightarrow$ $e_1=8V$, $e_2=10V$

$V_{oc}=e_2 \Rightarrow \boxed{V_{th}=10V}$

Then. Find i_{sc}.



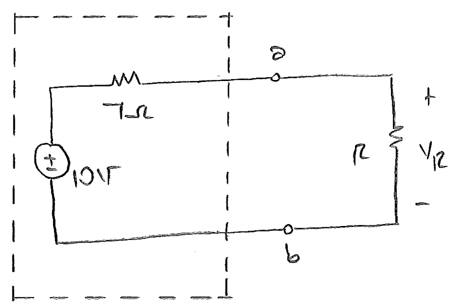
Node ①: $\frac{e_1-12}{8}+\frac{e_1}{12}+\frac{e_1}{12}=0$

$\Rightarrow \frac{7}{24}e_1=\frac{3}{2} \Rightarrow e_1=\frac{36}{7}V$

Node ⓐ: $i_{sc}=\frac{e_1}{12}+\frac{12}{12}=\frac{3}{7}+1=\frac{10}{7}$

$\Rightarrow \boxed{i_N=\frac{10}{7}A}$ $\Rightarrow R_{th}=\frac{V_{th}}{i_N}=\frac{10}{10/7}=\boxed{7\Omega}$

Finally.



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Thevenin equiv. seen by R

Want: $V_R=8V$.

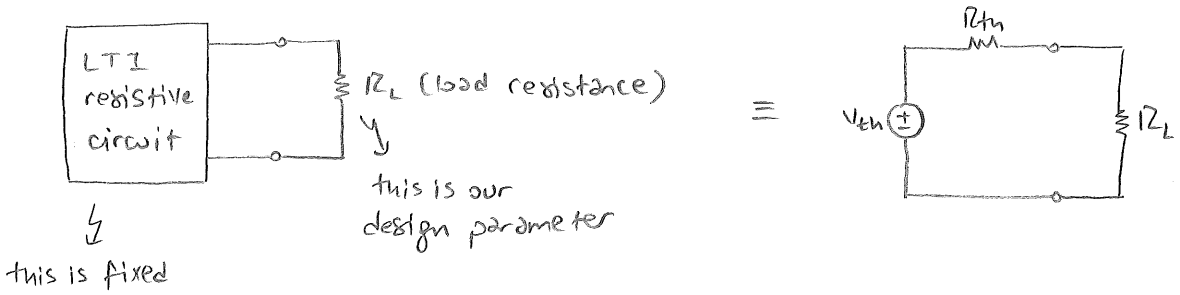
Design parameter: R

$V_R=\frac{R}{R+7}\cdot 10=8$

$\Rightarrow \boxed{R=28\Omega}$

Maximum Power Transfer

There are many applications in circuit theory where it is desirable to extract maximum possible power from a given circuit.



Now, the power delivered to the load is: $P_L = R_L \left[\dfrac{V_{th}}{R_{th} + R_L} \right]^2$

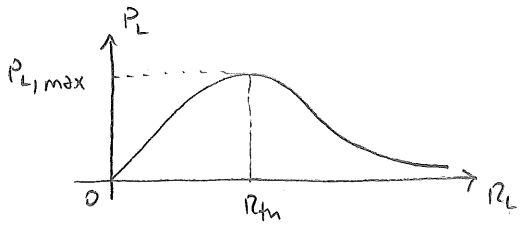
P_L: function of R_L (since V_{th} & R_{th} are fixed)

Question: $P_{L,max} = ?$

Answer: differentiate w.r.t. R_L.

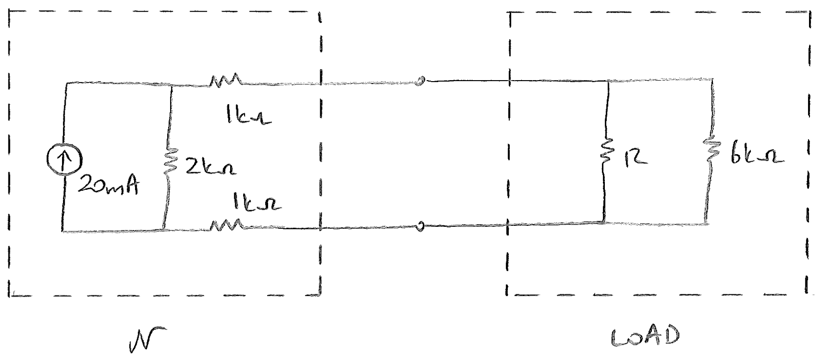
$$\frac{dP_L}{dR_L} = \frac{V_{th}^2}{(R_L + R_{th})^2} - 2R_L \frac{V_{th}^2}{(R_L + R_{th})^3} = V_{th}^2 \left\{ \frac{(R_L + R_{th}) - 2R_L}{(R_L + R_{th})^3} \right\} = V_{th}^2 \frac{R_{th} - R_L}{(R_L + R_{th})^3}$$

$$\frac{dP_L}{dR_L} = 0 \implies \boxed{R_L = R_{th}}$$



Conclusion: For max power transfer to the load, R_L must equal R_{th}!

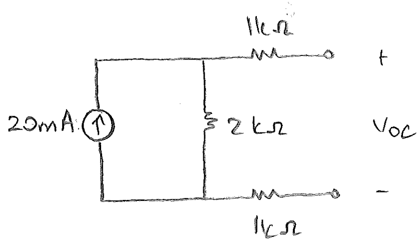
Example



a) Find R so that max power is delivered to the load.

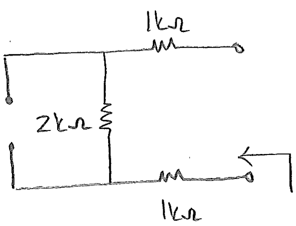
b) Find this max power.

Step1 Find the Thevenin equiv. of the circuit N.



$V_{oc} = (20mA)\times(2k\Omega) = 40V$

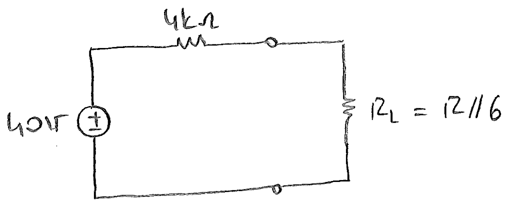
$\Rightarrow \boxed{V_{th} = 40V}$



$R_{th} = 1+2+1 = 4k\Omega$

$\Rightarrow \boxed{R_{th} = 4k\Omega}$

Step2 Study the equiv. circuit



For max power transfer we need $R_L = 4k\Omega$

$\Rightarrow \frac{1}{R} + \frac{1}{6} = \frac{1}{4} \quad \Rightarrow \boxed{R = 12k\Omega}$

$$P_{L,max} = R_L \left(\frac{40}{4k+R_L}\right)^2 \Bigg|_{R_L = 4k} = 4k\left(\frac{40}{8k}\right)^2 = (4k\Omega)\times(5mA)^2$$

$$= 4000 \times (5\times10^{-3})^2 = 0.1W$$

$\Rightarrow \boxed{P_{L,max} = 100mW}$