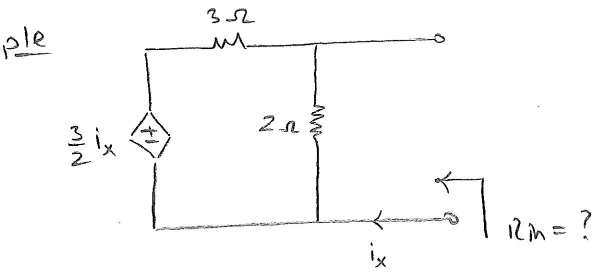
Input Resistances of LTI Resistive One-Ports

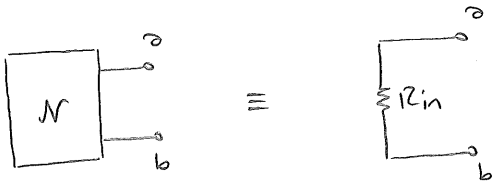
A one-port consisting only of ideal transformers, LTI resistors, and LTI dependent sources (whose control variables are within the one-port) is equivalent to a single LTI resistor.

Rin (input resistance)

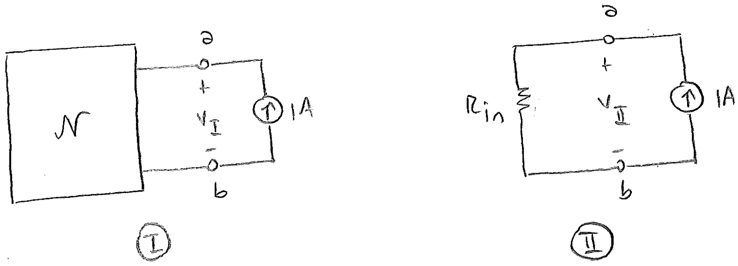
Example



Idea If



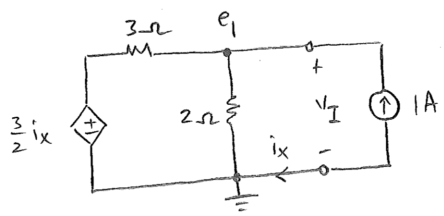
then we must have $v_I = v_{II}$ for the following configurations



$R_{in} = \frac{v_{II}}{1} = v_I$ → can be computed by solving the circuit in config. I

↓
unknown

Sol'n

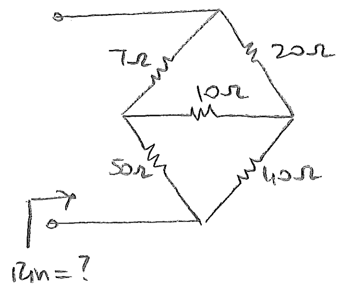


$$\frac{e_1 - \frac{3}{2} i_x}{3} + \frac{e_1}{2} - 1 = 0 \quad \& \quad i_x = -1A$$

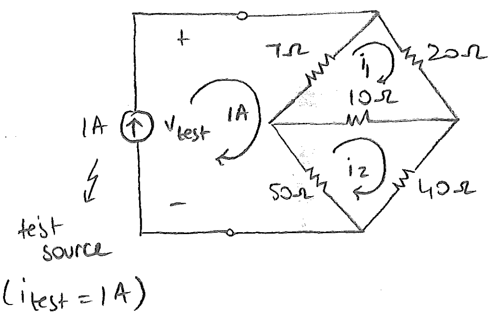
$$\Rightarrow \frac{e_1 + 3/2}{3} + \frac{e_1}{2} - 1 = 0 \Rightarrow \frac{5}{6} e_1 = \frac{1}{2}$$

$$\Rightarrow e_1 = \frac{3}{5} V \Rightarrow v_I = \frac{3}{5} V \Rightarrow \boxed{R_{in} = \frac{3}{5}\ \Omega}$$

Example



Sol'n Connect a test source



Mesh ①: $7(i_1-1)+20i_1+10(i_1-i_2)=0$

$\Rightarrow 37i_1-10i_2=7 \quad (1)$

Mesh ②: $50(i_2-1)+10(i_2-i_1)+40i_2=0$

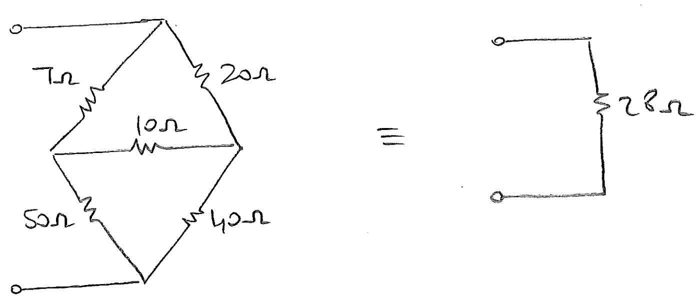
$\Rightarrow -10i_1+100i_2=50 \Rightarrow -i_1+10i_2=5 \quad (2)$

$(1)+(2) \Rightarrow 36i_1=12 \Rightarrow i_1=\frac{1}{3}\,A \overset{(2)}{\Rightarrow} i_2=\frac{8}{15}\,A$

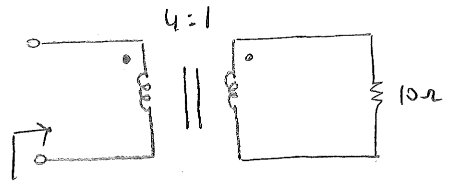
Now, $V_{test}=20i_1+40i_2=20\cdot\frac{1}{3}+40\cdot\frac{8}{15}=28\,V$

Then $R_{th}=\frac{V_{test}}{i_{test}}=\frac{28}{1}=\boxed{28\,\Omega}$

Hence

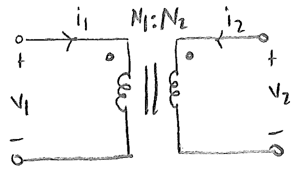


Example



$R_{in} = ?$

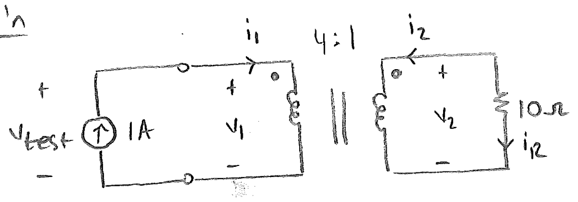
(Recall IT equations:



$$\frac{v_1}{N_1} = \frac{v_2}{N_2}$$

$$\& \quad N_1 i_1 + N_2 i_2 = 0$$

Sol'n



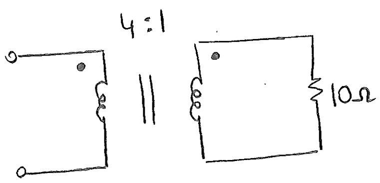
We have
$$\begin{cases} \frac{v_1}{4} = v_2 & (1) \\ 4i_1 + i_2 = 0 & (2) \end{cases}$$

Now, $i_1 = 1A \overset{(2)}{\Rightarrow} i_2 = -4A \Rightarrow i_R = 4A \Rightarrow v_2 = 10 i_R = 40V \overset{(1)}{\Rightarrow} v_1 = 160V$

$\Rightarrow v_{test} = 160V$

$\Rightarrow R_{in} = \frac{v_{test}}{i_{test}} = \frac{160}{1} = 160\Omega$

Here



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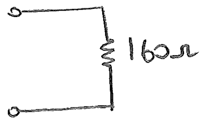
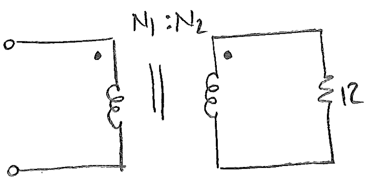


Exercise Show that in general we have



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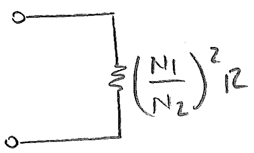


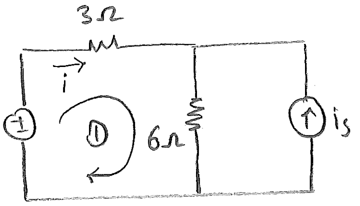
Linearity

Let $y(x_1, x_2, \dots, x_m)$ be a function of m variables $x_1, x_2, \dots, x_m$. This function is said to be linear if there exist real numbers $k_1, k_2, \dots, k_m$ such that

$y(x_1, x_2, \dots, x_m) = k_1 x_1 + k_2 x_2 + \dots + k_m x_m$.

In an LTI resistive circuit each branch current / voltage is a linear function of the voltages of the ind. voltage sources and the currents of the ind. current sources.

Example For the below circuit find k_1 and k_2 such that



$$i = k_1 v_s + k_2 i_s$$

KVL at ①: $-v_s + 3i + 6(i + i_s) = 0 \Rightarrow 9i = v_s - 6 i_s \Rightarrow i = \frac{1}{9} v_s - \frac{2}{3} i_s$

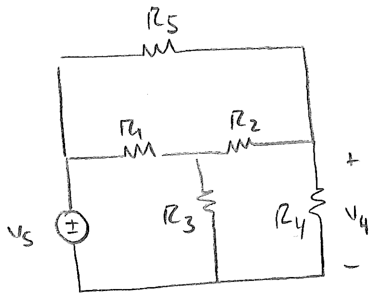
$\Rightarrow k_1 = \frac{1}{9}$ & $k_2 = -\frac{2}{3}$.

Example

It has been measured that $v_4 = 9V$ when $v_s = 24V$.

Find v_4 a) when $v_s = -8V$

b) when $v_s = 36V$



Sol'n By linearity $v_s = -8V \Rightarrow v_4 = -3V$

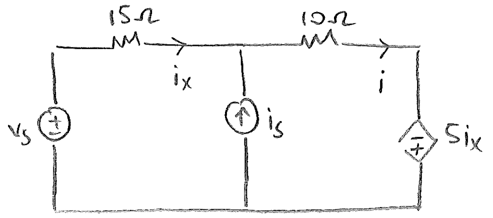
& $v_s = 36V \Rightarrow v_4 = 13.5V$

Linearity allows us to solve LTI resistive circuits by superposition

Example

$v_s = 20V$

$i_s = 4A$



Find i by superposition (i.e. consider one ind. source at a time)

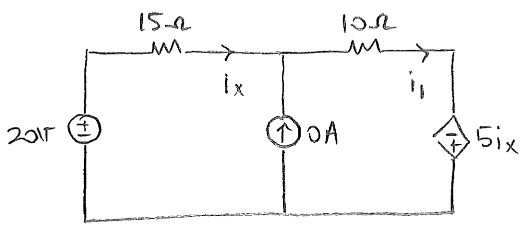
Sol'n Linearity implies $i = k_1 v_s + k_2 i_s$ (for some k_1, k_2)

$\Rightarrow i = k_1 \cdot 20 + k_2 \cdot 4 = \underbrace{[k_1 \cdot 20 + k_2 \cdot 0]}_{i_1} + \underbrace{[k_1 \cdot 0 + k_2 \cdot 4]}_{i_2}$

i_1 : the current passing through 10Ω resistor when $v_s = 20V$ and $i_s = 0$ (i.e. when ind. current source is killed)

i_2 : the current passing through 10Ω resistor when $i_s = 4A$ and $v_s = 0$ (i.e. when ind. voltage source is killed)

To compute i_1, kill the current source



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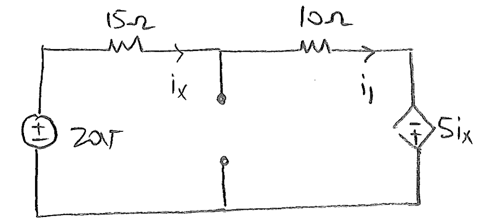

KVL : $-20 + 15i_x + 10i_1 - 5i_x = 0$ $(i_x = i_1)$

$\Rightarrow 20i_1 = 20$

$\Rightarrow \boxed{i_1 = 1A}$

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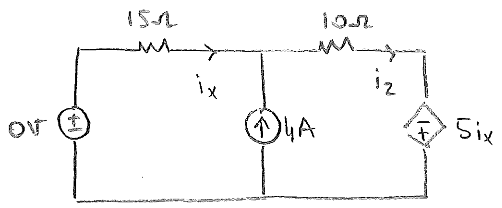
To compute i_2, kill the ind. voltage source



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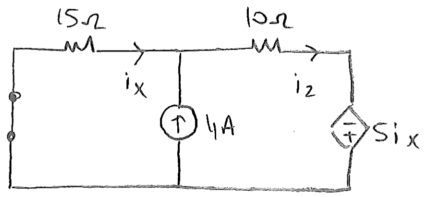


KVL : $15i_x + 10i_2 - 5i_x = 0 \quad (i_x = i_2 - 4)$

$\Rightarrow 20i_2 = 40$

$\Rightarrow \boxed{i_2 = 2A}$

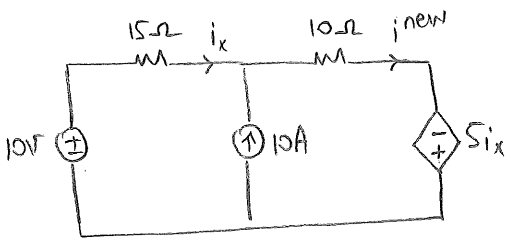
Finally, to find i we superpose i_1 and i_2 : $i = i_1 + i_2 = \boxed{3A}$

Remark killing an IVS means replacing it with short circuit.

killing an ICS means replacing it with open circuit.

Warning : DO NOT kill dependent sources!

Exercise Using your previous computations find i^{new}



Answer : $i^{new} = \underbrace{i_1^{new}}_{\frac{1}{2}i_1} + \underbrace{i_2^{new}}_{\frac{5}{2}i_2} = \frac{11}{2}A$