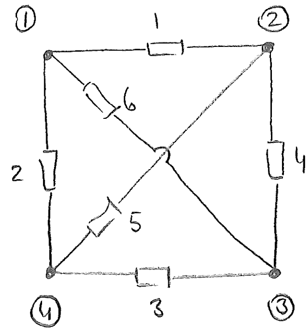
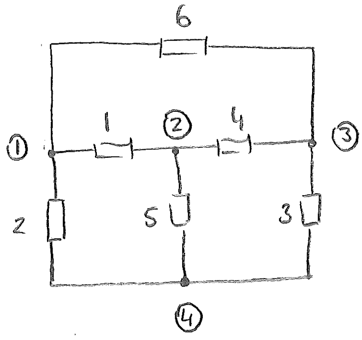
Definition A planar circuit is such that it can be drawn on plane in such a way that no two branches intersect at a point that is not a node.

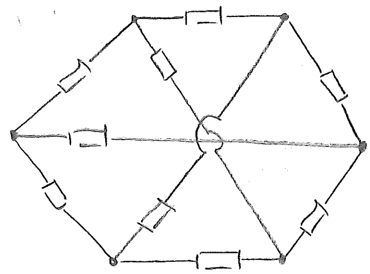
Ex :



planar because



whereas



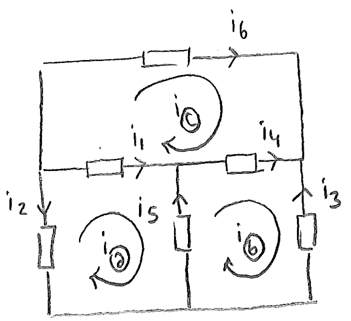
is a nonplanar circuit.

Planar circuits come with meshes and associated to each mesh there corresponds a mesh current. (Mesh current directions are chosen clockwise by convention.)

The relation of a branch current to mesh currents is as follows. Let the k^{th} branch touches the meshes (a) and (b). Suppose the direction of i_k is same with the mesh current i_a and opposite to the mesh current i_b. Then $i_k = i_a - i_b$.

Let the k^{th} branch touches only the mesh (a). Then $i_k = i_a$ if the directions are the same and $i_k = -i_a$ if opposite.

Ex :



Mesh currents : i_a, i_b, i_c

$i_1 = i_a - i_c$ $\qquad$ $i_4 = i_b - i_c$

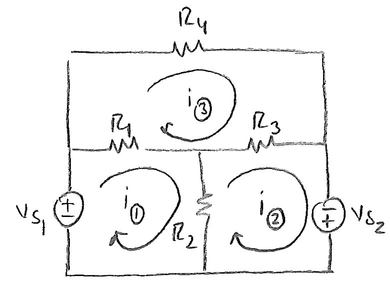
$i_2 = -i_a$ $\qquad$ $i_5 = i_b - i_a$

$i_3 = -i_b$ $\qquad$ $i_6 = i_c$

Mesh Current Analysis

The idea is to solve for mesh currents by writing KVL at each mesh.

Example



formulation variables i_1, i_2, i_3

KVL at mesh 1: $-V_{S_1} + V_{R_1} + V_{R_2} = 0$

$$R_1(i_1 - i_3) \leftarrow R_1 i_{R_1} \qquad R_2 i_{R_2} \rightarrow R_2(i_1 - i_2)$$

$$\Rightarrow -V_{S_1} + R_1(i_1 - i_3) + R_2(i_1 - i_2) = 0 \qquad (1)$$

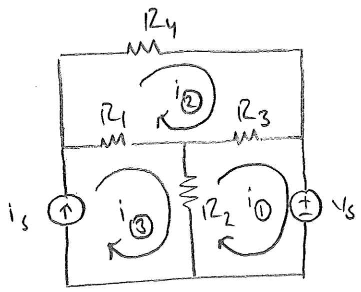
KVL at mesh 2: $-V_{S_2} + R_2(i_2 - i_1) + R_3(i_2 - i_3) = 0 \qquad (2)$

KVL at mesh 3: $R_4 i_3 + R_3(i_3 - i_2) + R_1(i_3 - i_1) = 0 \qquad (3)$

$$(1),(2),(3) \Rightarrow \underbrace{\begin{bmatrix} R_1+R_2 & -R_2 & -R_1 \\ -R_2 & R_2+R_3 & -R_3 \\ -R_1 & -R_3 & R_1+R_3+R_4 \end{bmatrix}}_{Z_M\text{ : mesh impedance matrix}} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}}_{i_M} = \underbrace{\begin{bmatrix} V_{S_1} \\ V_{S_2} \\ 0 \end{bmatrix}}_{V_S}$$

Mesh Analysis with current sources

Example



Note that $i_3 = i_s$, therefore known.

Formulation var. : i_1, i_2

KVL at ①: $R_2(i_{①} - i_s) + R_3(i_{①} - i_{②}) + v_s = 0$

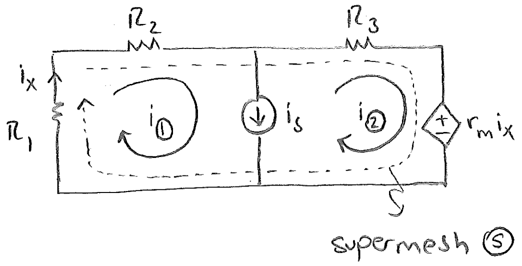
KVL at ②: $R_1(i_{②} - i_s) + R_4 i_{②} + R_3(i_{②} - i_{①}) = 0$

$$\begin{bmatrix} R_2+R_3 & -R_3 \\ -R_3 & R_1+R_3+R_4 \end{bmatrix}\begin{bmatrix} i_{①} \\ i_{②} \end{bmatrix} = \begin{bmatrix} R_2 i_s - v_s \\ R_1 i_s \end{bmatrix}$$

Modified Mesh Analysis

When there is a current source between two meshes we cannot express its voltage in terms of mesh currents. In such a case we write KVL pretending that the two meshes (between which there is a current source) are a single mesh. Such mesh is called a <u>supermesh</u>.

Example



formulation var. : $i_{①}, i_{②}$

KVL at supermesh Ⓢ : $(R_1 + R_2) i_{①} + R_3 i_{②} + r_m i_x = 0$, $\quad i_x = i_{①}$ $\quad \Big\}\ (R_1 + R_2 + r_m) i_{①} + R_3 i_{②} = 0 \quad (1)$

constraint eqn. : $i_{①} - i_{②} = i_s \quad (2)$

(1) & (2) $\Rightarrow$
$$\begin{bmatrix} R_1+R_2+r_m & R_3 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} i_{①} \\ i_{②} \end{bmatrix} = \begin{bmatrix} 0 \\ i_s \end{bmatrix} \quad (3)$$

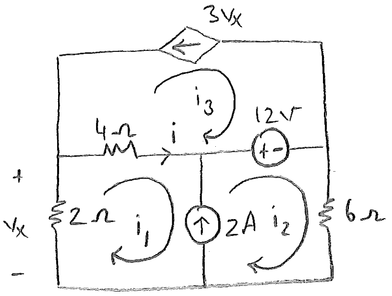
Let $R_1 = R_2 = r_m = 1\Omega$ & $i_s = 1A$. Find $i_{①}, i_{②}$.

(3) $\Rightarrow$
$$\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} i_{①} \\ i_{②} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} i_{①} \\ i_{②} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -1 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/4 \\ -3/4 \end{bmatrix}$$

$\Rightarrow i_{①} = \frac{1}{4} A$ & $i_{②} = -\frac{3}{4} A$

Example



Find i.

Mesh current formulation

Supermesh ①+②: $2i_1 + 4(i_1 - i_3) + 12 + 6i_2 = 0$

$\Rightarrow 6i_1 + 6i_2 - 4i_3 = -12 \quad (1)$

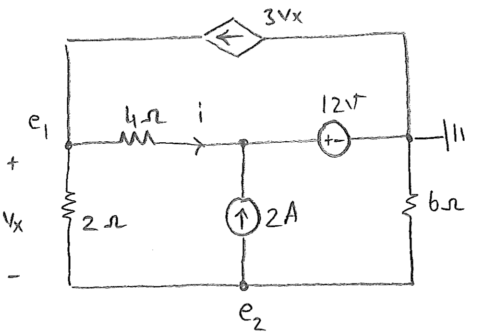
Constraint 1: $\left.\begin{array}{l} i_3 = -3V_x \\ V_x = -2i_1 \end{array}\right\} \quad i_3 = 6i_1 \quad (2)$

Constraint 2: $i_2 - i_1 = 2 \Rightarrow i_2 = i_1 + 2 \quad (3)$

(1),(2),(3) $\Rightarrow 6i_1 + 6(i_1+2) - 4(6i_1) = -12 \Rightarrow -12i_1 = -24 \Rightarrow i_1 = 2A \quad (4)$

(2) & (4) $\Rightarrow i = i_1 - i_3 = i_1 - 6i_1 = -5i_1 = \boxed{-10A}$

Node voltage formulation



Node ①: $\frac{e_1 - e_2}{2} + \frac{e_1 - 12}{4} - 3V_x = 0$ & $V_x = e_1 - e_2$

$\Rightarrow \left(\frac{1}{2} + \frac{1}{4} - 3\right)e_1 + \left(-\frac{1}{2} + 3\right)e_2 = 3$

$\Rightarrow -\frac{9}{4}e_1 + \frac{5}{2}e_2 = 3 \quad (1)$

Node ②: $2 + \frac{e_2 - e_1}{2} + \frac{e_2}{6} = 0$

$\Rightarrow -\frac{1}{2}e_1 + \frac{2}{3}e_2 = -2 \quad (2)$

$$(1)\&(2) \Rightarrow \underbrace{\begin{bmatrix} -\frac{9}{4} & \frac{5}{2} \\ -\frac{1}{2} & \frac{2}{3} \end{bmatrix}}_{Y} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \Rightarrow \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \underbrace{\frac{1}{-\frac{3}{2}+\frac{5}{4}} \begin{bmatrix} \frac{2}{3} & -\frac{5}{2} \\ \frac{1}{2} & -\frac{9}{4} \end{bmatrix}}_{Y^{-1}} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -28 \\ -24 \end{bmatrix}$$

Hence, $i = \frac{e_1 - 12}{4} = \frac{-28-12}{4} = \boxed{-10A}$

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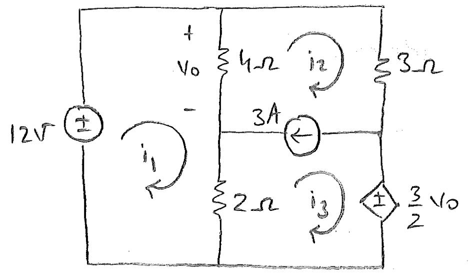
Example



write mesh equations in matrix form & find V_o.

Mesh ①: $-12 + 4(i_1 - i_2) + 2(i_1 - i_3) = 0 \Rightarrow 6i_1 - 4i_2 - 2i_3 = 12 \quad (1)$

Supermesh ②+③: $\left.\begin{array}{l} 4(i_2 - i_1) + 3i_2 + \frac{3}{2}V_o + 2(i_3 - i_1) = 0 \\ V_o = 4(i_1 - i_2) \end{array}\right\}$

$\Rightarrow 4(i_2 - i_1) + 3i_2 + 6(i_1 - i_2) + 2(i_3 - i_1) = 0 \Rightarrow i_2 + 2i_3 = 0 \quad (2)$

Constraint: $i_2 - i_3 = 3 \quad (3)$

$$(1),(2),(3) \Rightarrow \begin{bmatrix} 6 & -4 & -2 \\ 0 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 3 \end{bmatrix}$$

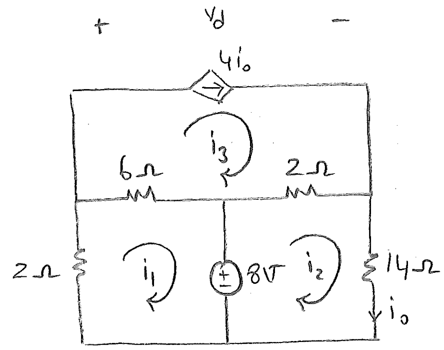
$(3) \Rightarrow \quad i_2 = i_3 + 3 \quad (4)$

$(2) \& (4) \Rightarrow (i_3 + 3) + 2i_3 = 0 \Rightarrow i_3 = -1A \overset{(4)}{\Rightarrow} i_2 = 2A$

$(1) \Rightarrow 6i_1 - 4(2) - 2(-1) = 12 \Rightarrow i_1 = 3A$

$\Rightarrow V_o = 4(i_1 - i_2) = 4(3-2) = \boxed{4V}$

Example



Find the power supplied /absorbed by the dependent source.

Mesh ①: $2i_1 + 6(i_1 - i_3) + 8 = 0 \Rightarrow 8i_1 - 6i_3 = -8$ (1)

Mesh ②: $-8 + 2(i_2 - i_3) + 14i_2 = 0 \Rightarrow 16i_2 - 2i_3 = 8$ (2)

Constraint: $\left.\begin{aligned} i_3 &= 4i_o \\ i_o &= i_2 \end{aligned}\right\}$ $i_3 = 4i_2$ (3)

(2)&(3) $\Rightarrow 16i_2 - 2(4i_2) = 8 \Rightarrow i_2 = 1A \overset{(3)}{\Rightarrow} i_3 = 4A$

(1) $\Rightarrow 8i_1 = 6i_3 - 8 = 6(4) - 8 = 16 \Rightarrow i_1 = 2A$

KVL at outer loop: $V_d + 14i_2 + 2i_1 = 0 \Rightarrow V_d = -14i_2 - 2i_1 = -14 - 4 = -18V$

$\Rightarrow P_\diamond = V_d \cdot 4i_o = -18 \cdot 4(1) = -72W$

$\Rightarrow$ Dep. source supplies 72W power.