#1 29 Eylül 2023

Ch. I

## Basic Concepts

**Charge** : A fundamental conserved property of subatomic particles. Measured in Coulombs (C) For instance, the electron has $-1e$ charge, the proton $+1e$. ($e$ : elementary charge unit. $1e \cong 1.602 \times 10^{-19}$ C) We can talk about the net charge of arbitrary matter.

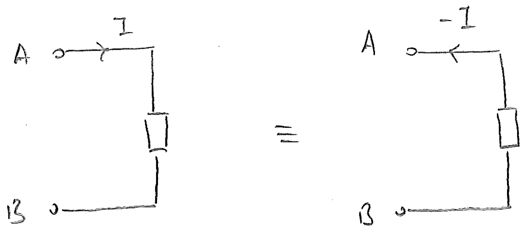
Ex Block $\boxed{\begin{matrix} n^0 & p^+ & e^- \\ n^0 & p^+ & \end{matrix}}$ has net charge of $1e$.

Charge is denoted by $q$ or $Q$.

**Current** Flow rate of (net) charge through a given area. Measured in Amperes (A). ($1\,A = 1\,C/s$) Denoted by $i$ or $I$.

$$\boxed{i(t) = \frac{dq(t)}{dt}} \implies q(t) = q(0) + \int_0^t i(\tau)\,d\tau$$

To be able to talk about current we need a specified direction.

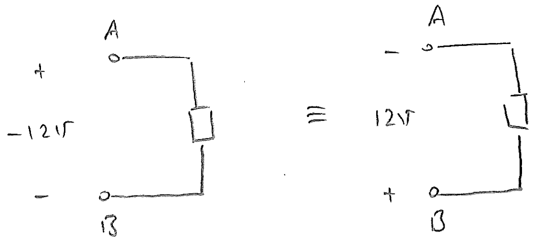



**Voltage** Consider two points $x$ & $y$. The work done against the electrical field to move a unit charge (1C) from $y$ to $x$ is the voltage $V_{xy}$. This work is independent of the path through which the charge travels and depends only on the endpoints $x$ & $y$. Therefore $V_{xy} = -V_{yx}$. Voltage is measured in Volts (V). ($1V = 1J/C$)

$$\boxed{V = \frac{dW}{dq}}$$ ($W$ : work, energy)

Ex Let $V_{AB} = -12V$

To be able to talk about voltage we need a specified polarity.

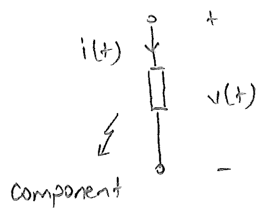


**Ground** Common reference point for voltage measurements in a circuit. Given some point $x$ in a circuit, by $V_x$ we mean the voltage between the point $x$ and ground, i.e. $V_{xg}$. Symbol for ground : $\perp$

**Power** Rate of change of energy. Denoted by $P$ or $p$. Measured in Watts (W) (1W = 1 J/s)

$$p = \frac{dw}{dt} = \underbrace{\frac{dw}{dq}}_{v} \cdot \underbrace{\frac{dq}{dt}}_{i} \Rightarrow \boxed{p(t) = v(t)\, i(t)}$$

**Passive sign convention** We adopt the convention that, given an electrical component, the polarity of voltage and the direction of current are chosen such that the current enters (the component) from the "+" labeled terminal and leaves from the "−" labeled terminal.




According to our convention:

if $p(t) = i(t)v(t) > 0$ then the component is absorbing electrical power. E.g. heater.

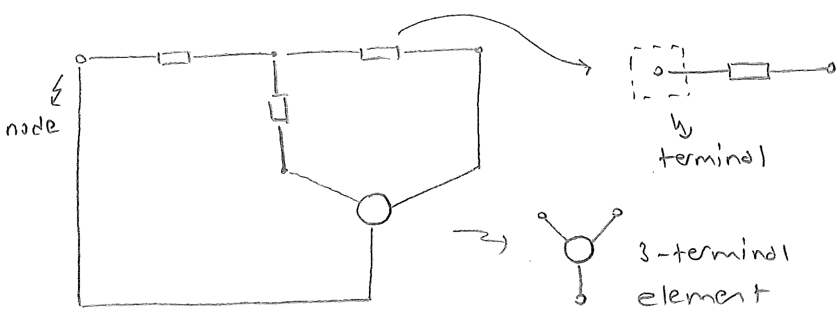
if $p(t) = i(t)v(t) < 0$ then the component is delivering electrical power. E.g. battery.

**Assumptions** In this course we make the following assumptions

(A1) Electrical effects happen instantaneously throughout the circuit.

(A2) The net charge inside any closed surface is always zero.

**Electric circuit** Interconnection of (basic) electrical components.



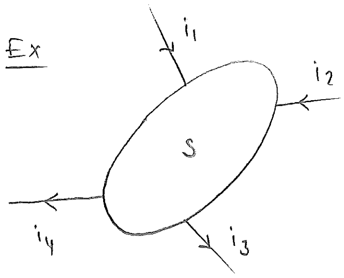

2-terminal element (single branch element)

node : point where two or more terminals meet.

# Kirchhoff's Laws

**Kirchhoff's Current Law (KCL)** [ A direct implication of our assumption A2: " The net charge inside any closed surface is always zero." ] For any closed surface the sum of incoming currents equals the sum of outgoing currents.

Ex



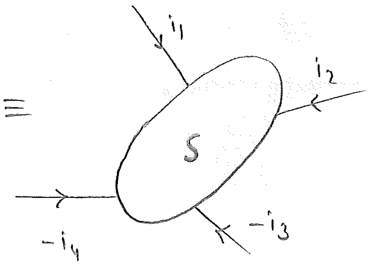

incoming : $i_1, i_2$

outgoing : $i_3, i_4$

KCL : $\Sigma$ incoming $=$ $\Sigma$ outgoing

$$\Rightarrow \quad i_1 + i_2 = i_3 + i_4$$

$\equiv$

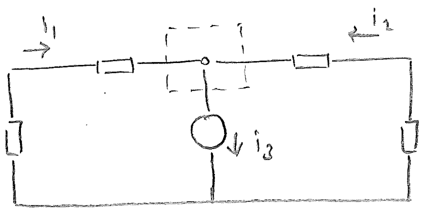



incoming : $i_1, i_2, -i_3, -i_4$

outgoing : none

KCL $\Rightarrow$ $i_1 + i_2 + (-i_3) + (-i_4) = 0$

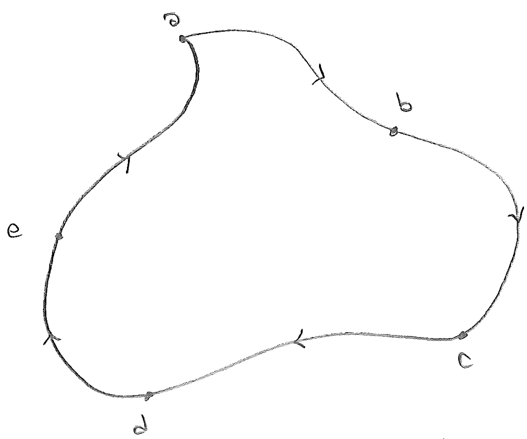
Ex




KCL $\Rightarrow$ $i_1 + i_2 = i_3$

**Kirchhoff's Voltage Law (KVL)** [ A direct implication of path independence for the potential energy difference between two points. ] Sum of voltages along any loop (closed path) is zero.

Ex



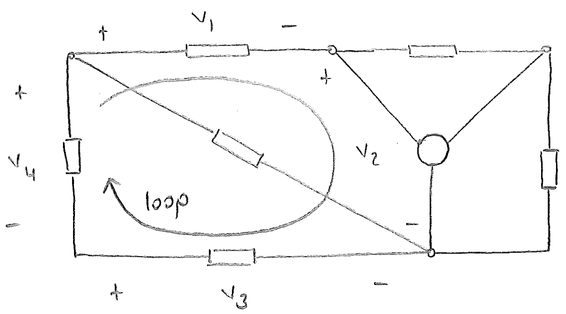

$V_{ab} + V_{bc} + V_{cd} + V_{de} + V_{ea} = V_{aa} = 0$

Likewise, $V_{ae} + V_{ed} + V_{dc} + V_{cb} + V_{ba} = 0$

Also, $V_{ab} + V_{bc} = V_{ae} + V_{ed} + V_{dc}$

and so on.

Ex




KVL $\Rightarrow$ $v_1 + v_2 - v_3 - v_4 = 0$

let

$$\left.\begin{aligned} v_1(t) &= 2\cos\left(10t + \frac{\pi}{4}\right)\ V \\ v_2(t) &= 12\ V \\ v_3(t) &= 5\sin(5t)\ V \end{aligned}\right\}\ v_4(t) = ?$$

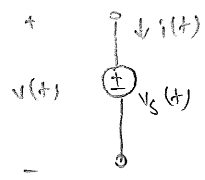
KVL $\Rightarrow$ $v_4(t) = v_1(t) + v_2(t) - v_3(t)$

$\Rightarrow v_4(t) = 12 + 2\cos\left(10t + \frac{\pi}{4}\right) - 5\sin(5t)\ V$

## Terminal Equations

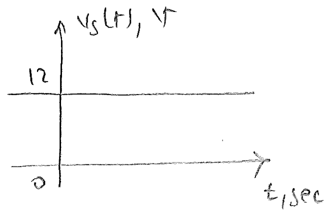
A circuit element is characterized by well-defined relations satisfied by the current(s) through the element and the voltage(s) across its terminals. Such relations are called terminal equations. Terminal equations depend only on the element and not on the rest of the circuit that the element is part of.

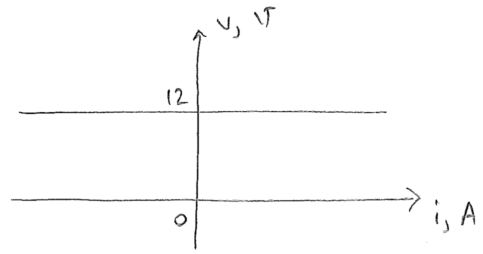
Independent voltage source (IVS) A two-terminal (single branch) element with branch relation $v(t) = v_s(t)$, i.e., the voltage of IVS is independent of the current $i(t)$ passing through it.




schematic representation

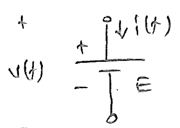
Ex



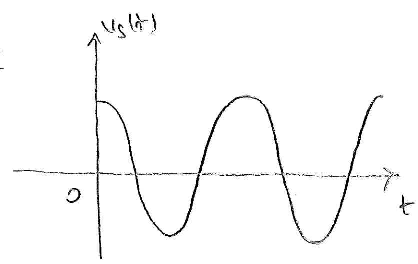

A constant voltage source (sometimes called battery)

A battery is depicted by

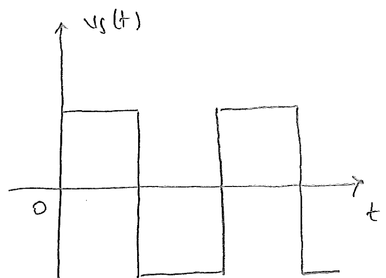



$v(t) = E$ (constant)

Ex

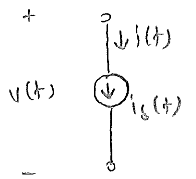



sinusoidal voltage




square

<u>Independent current source (ICS)</u> A single branch element with branch relation $i(t) = i_s(t)$, i.e., the current of ICS is independent of the voltage across its terminals.

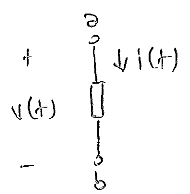



<u>Ex</u> $i_s(t) = I_m \cos(\omega t + \phi)$ sinusoidal current source (AC source)

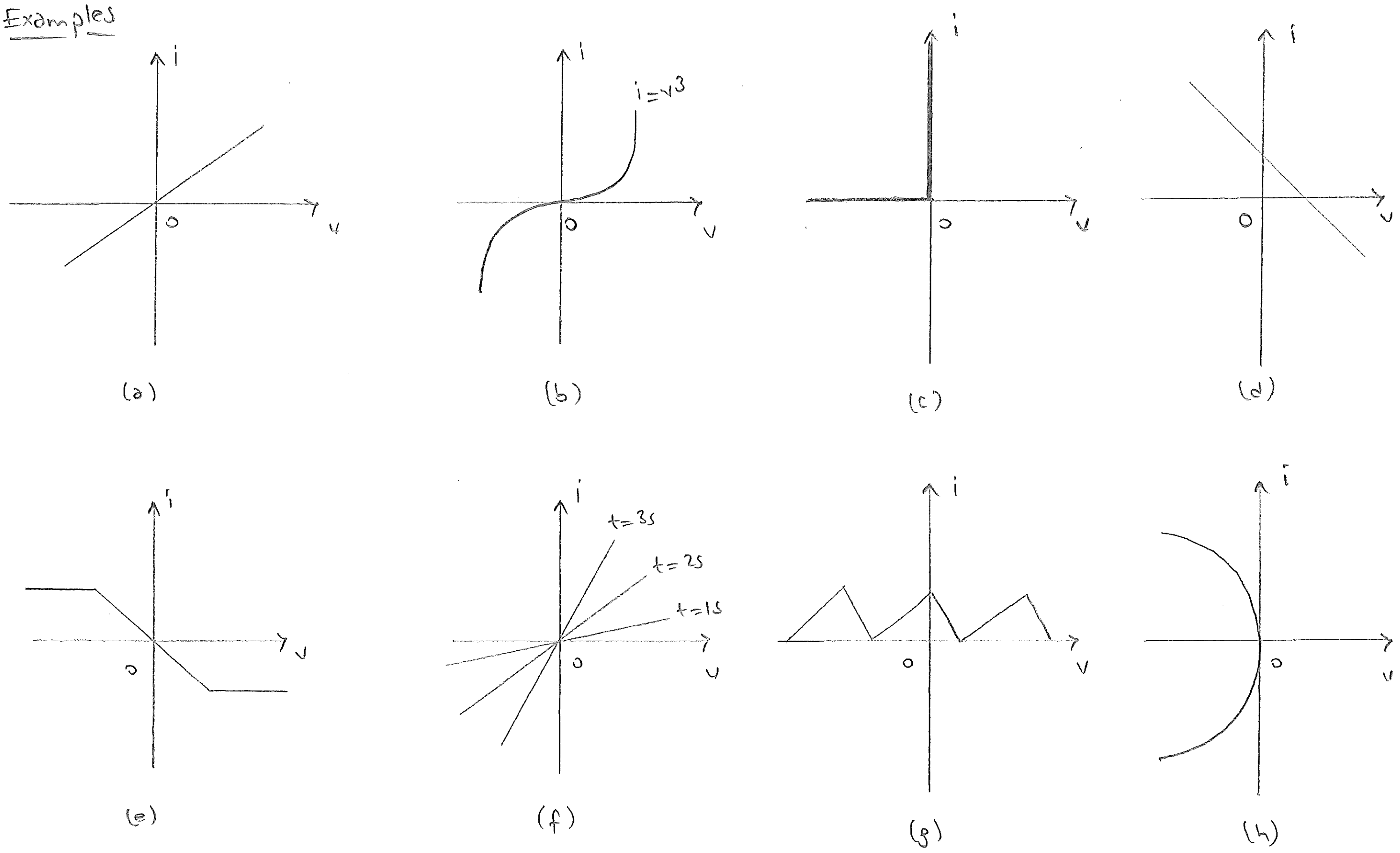
$i_s(t) = I_0$ constant current source (DC source)

<u>Remark</u> IVS & ICS are called <u>active</u> elements because they are capable of delivering arbitrary amount of power to the circuit they are connected to throughout intervals of arbitrary length.

<u>Resistor</u> General name for a 2-terminal element whose branch current & branch voltage satisfy a relation described by a curve on the i-v plane.




<u>Examples</u>



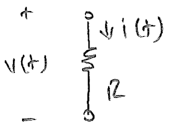
**Linear** Variables $x, y$ are said to satisfy a linear relation if there exist constants $a, b$ (not both $a$ and $b$ are zero) such that $ax + by = 0$.

**Bilateral** Bilateral elements have i-v curves that are symmetric w.r.t. the origin. When we are to connect a bilateral element between two nodes it is immaterial which terminal is connected to which node. For nonbilateral elements this is not so.

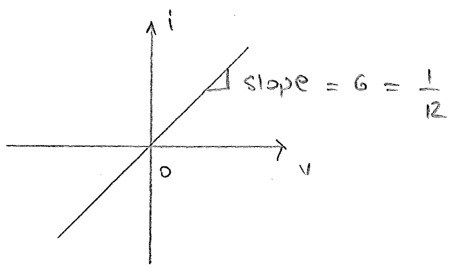
**Active** A resistor is said to be active if its i-v curve contains a point $(v_0, i_0)$ satisfying $v_0 i_0 < 0$. In other words, active $\Leftrightarrow$ i-v curve visits 2nd or 4th quadrant.

| | linear / nonlinear | voltage-controlled / current-controlled | time-varying / time-invariant | bilateral / nonbilateral | active / passive |
|---|---|---|---|---|---|
| a | L | VC & CC | TI | B | P |
| b | N | VC & CC | TI | B | P |
| c | N | neither | TI | N | P |
| d | N | VC & CC | TI | N | A |
| e | N | VC | TI | B | A |
| f | L | VC & CC | TV | B | P |
| g | N | VC | TI | N | A |
| h | N | CC | TI | N | A |

**LTI (linear time-invariant) resistor**




$$\boxed{v(t) = R\, i(t)}$$ (Ohm's Law)



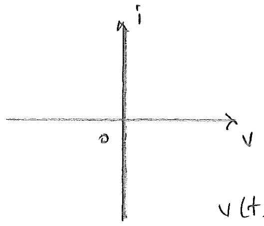

R: resistance, measured in Ohms ($\Omega$)

G: conductance, measured in Siemens (S) or Mhos ($\mho$)

An LTI resistor is both voltage & current controlled for all $R$ except

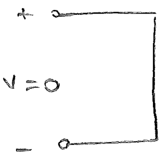
$R = 0$ & $R = \infty$ $(G = 0)$

$\underline{R=0}$



$v(t) = 0$

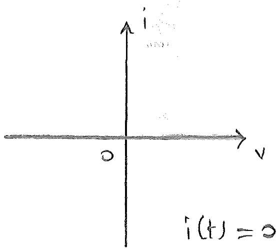
symbol :



$v = 0$ $\downarrow i(t)$ can be anything

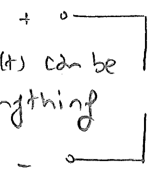
this case is called "short circuit"

$\underline{R=\infty}$



$i(t) = 0$

symbol :



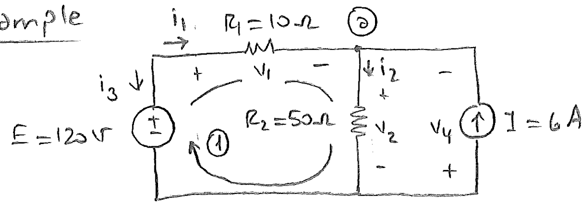
$v(t)$ can be anything $\downarrow i = 0$

this case is called "open circuit"

$\underline{\text{power}}$ :

$$\left.\begin{aligned} p: vi &= v \cdot \frac{v}{R} = \frac{1}{R} v^2 \\ &= Ri \cdot i = Ri^2 \end{aligned}\right\} \quad \boxed{p = \frac{1}{R} v^2 = Ri^2} \quad \text{for LTI resistor.}$$

$\underline{\text{Example}}$




a) Find $i_1, i_2, v_1, v_2$

b) Verify that total generated power equals total dissipated power.

$\underline{\text{Sol'n}}$ a) Ohm's Law : $v_1 = R_1 i_1 = 10 i_1$ & $v_2 = R_2 i_2 = 50 i_2$

KCL at node (a) : $i_1 - i_2 + I = 0 \Rightarrow i_1 - i_2 = -6$ (1)

KVL at loop (1) : $v_1 + v_2 - E = 0 \Rightarrow v_1 + v_2 = 120 \Rightarrow 10 i_1 + 50 i_2 = 120$ (2)

(1) & (2) $\Rightarrow$ $\boxed{i_1 = -3A, \; i_2 = 3A}$ $\Rightarrow$ $\boxed{v_1 = -30V, \; v_2 = 150V}$

b) voltage source : $i_3 = -i_1 = 3A \Rightarrow P_E = 3 \cdot 120 = 360W$ (absorbing)

current source : $v_4 = -v_2 = -150V \Rightarrow P_I = 6 \cdot (-150) = -900W$ (delivering)

$R_1$ : $P_{R_1} = i_1 v_1 = 90W$ (absorbing)

$R_2$ : $P_{R_2} = R_2 i_2^2 = 450W$ (absorbing)

Hence,

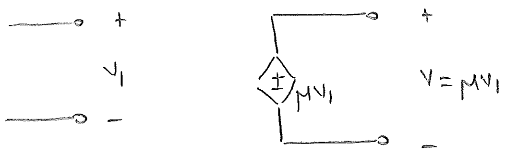
power generated $= |-900| = \underline{900W}$ by ICS

power dissipated :

$= 360 + 90 + 450 = \underline{900W}$

**Dependent source**: A linear dependent source has terminal characteristics controlled by a current or a voltage of some other branch in the circuit.

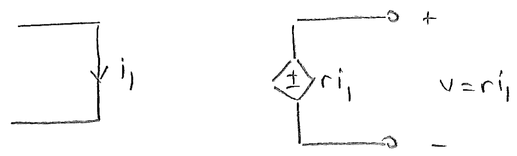
1| voltage controlled voltage source (VCVS)




$v_1$ : control variable

$\mu$ : voltage gain

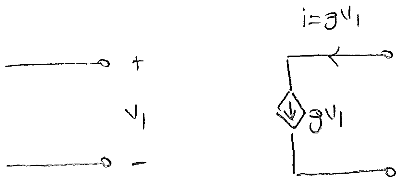
2| current controlled voltage source (CCVS)




$i_1$ : control var.

$r$ : transresistance

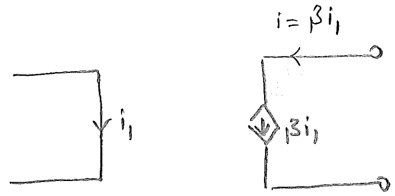
3| VCCS




$v_1$ : control var.

$g$ : transconductance

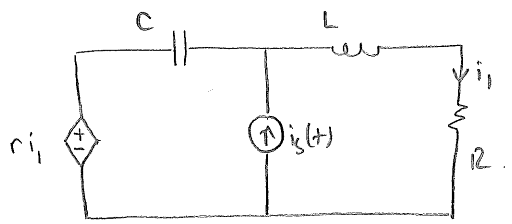
4| CCCS



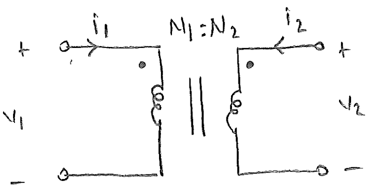

$i_1$ : control var.

$\beta$ : current gain

Ex




**Ideal transformer**  A 2-branch IT is depicted by




The branch relations: $\boxed{\frac{v_1}{N_1} = \frac{v_2}{N_2}}$ & $\boxed{N_1 i_1 + N_2 i_2 = 0}$

$N_k$ is the number of turns for the $k^{th}$ branch $k = 1, 2$

Note that the total power of IT is $i_1 v_1 + i_2 v_2 = 0$ (Exercise : prove this)