EE209 Second Midterm Exam 21 Dec 2023

First name:_____

Last name: Soln

Student ID:_____

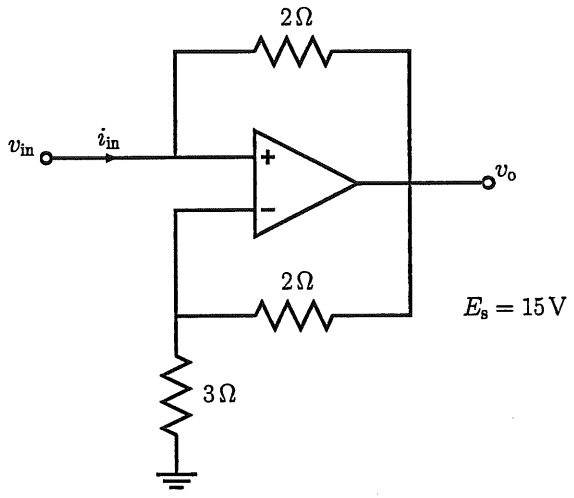
Signature:_____

Read before you start:

- There are four questions.
- The examination is closed-book.
- No computer/calculator is allowed.
- The duration of the examination is 100 minutes.
- Besides correctness, the CLARITY of your presentation will also be graded.

Q1	Q2	Q3	Q4	Total

Q1.



Find and sketch the transfer (v_{in} - v_o) and input (v_{in} - i_{in}) characteristics.

$v_+ = v_{in}$ (1)

$\frac{v_-}{3} + \frac{v_- - v_o}{2} = 0 \Rightarrow v_- = \frac{3}{5} v_o$ (2)

$i_{in} = \frac{v_{in} - v_o}{2}$ (3)

linear $v_+ = v_-$ & $-15 \le v_o \le 15$

(1) & (2) $\Rightarrow$ $v_{in} = \frac{3}{5} v_o \Rightarrow \boxed{v_o = \frac{5}{3} v_{in}}$

$\Rightarrow -15 \le \frac{5}{3} v_{in} \le 15 \Rightarrow \boxed{-9 \le v_{in} \le 9}$

(3) $\Rightarrow i_{in} = \frac{1}{2} v_{in} - \frac{1}{2} v_o = \frac{1}{2} v_{in} - \frac{1}{2}\left(\frac{5}{3} v_{in}\right)$

$\Rightarrow \boxed{i_{in} = -\frac{1}{3} v_{in}}$

+sat $v_+ > v_-$ & $\boxed{v_o = 15\,V}$

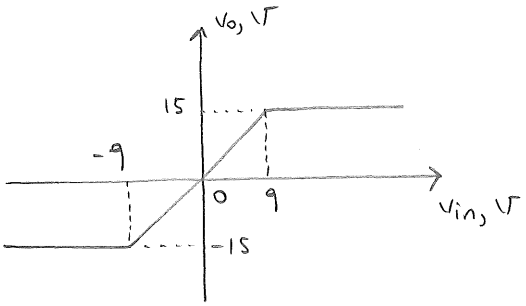
(1) & (2) $\Rightarrow v_{in} > \frac{3}{5}(15) \Rightarrow \boxed{v_{in} > 9}$

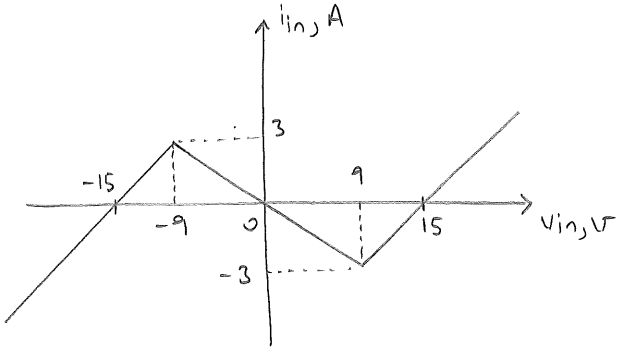
(3) $\Rightarrow \boxed{i_{in} = \frac{1}{2}(v_{in} - 15)}$

-sat $v_+ < v_-$ & $\boxed{v_o = -15\,V}$

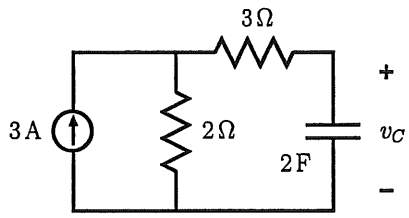
(1) & (2) $\Rightarrow v_{in} < \frac{3}{5}(-15) \Rightarrow \boxed{v_{in} < -9}$

(3) $\Rightarrow \boxed{i_{in} = \frac{1}{2}(v_{in} + 15)}$





Q2.

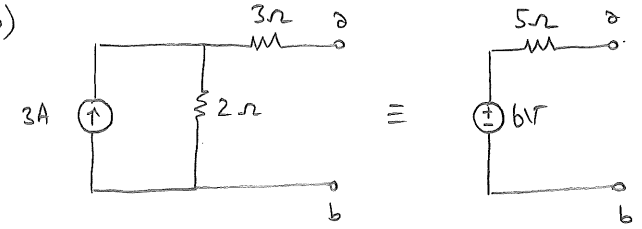


$v_C(0) = 2\text{V}$

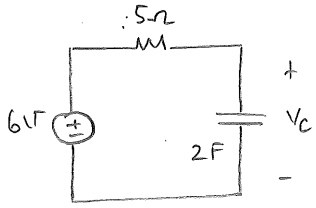
(a) Find $v_C(t)$ for $t \geq 0$.

(b) Find the energy stored in the capacitor in the DC steady state.

a)



Hence we can work with



$v_c(0) = 2\text{V}$

$6 = 5i_c + v_c = 5(2\dot{v}_c) + v_c$

$\Rightarrow \quad \dot{v}_c + \frac{1}{10} v_c = \frac{3}{5}$

$\Rightarrow \quad v_c(t) = 6 + Ke^{-\frac{t}{10}}$

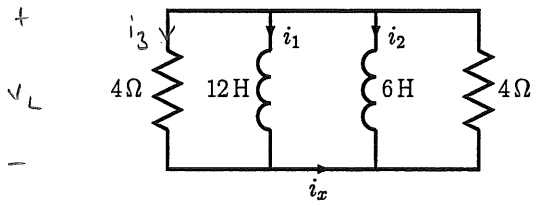
$v_c(0) = 2 \Rightarrow 6 + K = 2 \Rightarrow K = -4$

$\Rightarrow \boxed{v_c(t) = 6 - 4e^{-\frac{t}{10}}\text{V}}$

b) $v_c(\infty) = 6\text{V}$

$\Rightarrow E_c = \frac{1}{2} C v_c(\infty)^2 = \frac{1}{2} \cdot 2 \cdot 6^2 = \boxed{36\text{J}}$

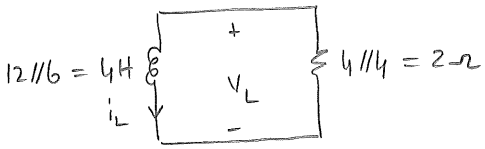
Q3.



$i_1(0) = 5\,\text{A}$
$i_2(0) = 1\,\text{A}$

Find $i_x(t)$ for $t \geq 0$.

Equivalent circuit :



$$i_L(0) = i_1(0) + i_2(0) = 6\,\text{A}$$

$$\tau = \frac{L}{R} = \frac{4}{2} = 2\,\text{sec}$$

$$\Rightarrow i_L(t) = i_L(0)\, e^{-t/\tau} = 6e^{-t/2}\,\text{A}$$

$$\Rightarrow v_L(t) = L\frac{d}{dt} i_L(t) = 4\frac{d}{dt}\left\{6e^{-t/2}\right\}$$

$$= -12e^{-t/2}\,\text{V}$$

$$i_1(t) = i_1(0) + \frac{1}{12}\int_0^t v_L(\alpha)\,d\alpha$$

$$= 5 + \frac{1}{12}\int_0^t -12e^{-\alpha/2}\,d\alpha$$

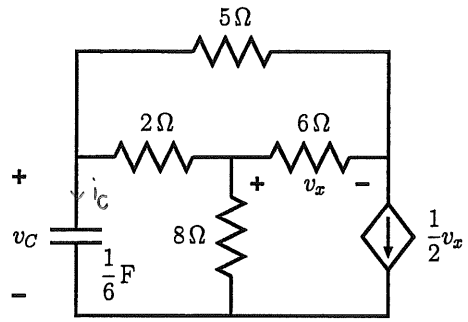
$$= 5 + 2e^{-\alpha/2}\Big|_0^t = 5 + 2\left(e^{-t/2} - 1\right)$$

$$= 3 + 2e^{-t/2}\,\text{A}$$

$$i_3(t) = \frac{v_L(t)}{4} = -3e^{-t/2}\,\text{A}$$

$$i_x(t) = i_1(t) + i_3(t) = \boxed{3 - e^{-t/2}\,\text{A}}$$

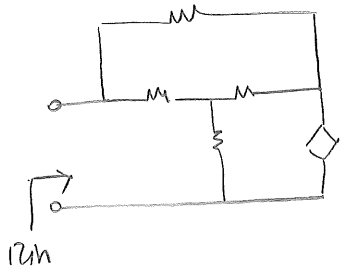
Q4.



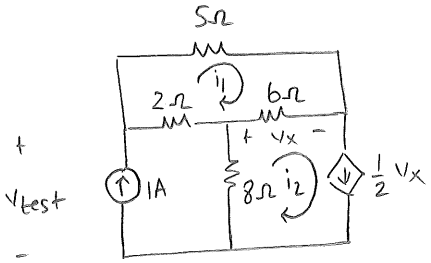
$v_C(0) = 12\,\text{V}$

Find $v_x(t)$ for $t \geq 0$.

First, find R_{in} :



Let's connect a test source :



(*)

Mesh ① : $2(i_1 - 1) + 5i_1 + 6(i_1 - i_2) = 0$

$\Rightarrow 13i_1 - 6i_2 = 2 \quad (1)$

Constraint : $i_2 = \frac{1}{2}v_x = \frac{1}{2}\{6(i_2 - i_1)\} = 3i_2 - 3i_1$

$\Rightarrow 2i_2 - 3i_1 = 0 \quad (2)$

(1) & (2) $\Rightarrow i_1 = \frac{1}{2}\,A$ & $i_2 = \frac{3}{4}\,A$

$\Rightarrow v_{test} = 2(1 - i_1) + 8(1 - i_2) = 2\left(\frac{1}{2}\right) + 8\left(\frac{1}{4}\right) = 3\,V$

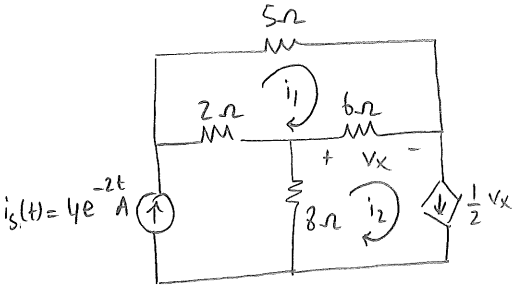
$\Rightarrow R_{in} = \frac{v_{test}}{i_{test}} = \frac{3}{1} = 3\Omega$

$\Rightarrow \tau = R_{in}C = 3 \cdot \frac{1}{6} = \frac{1}{2}\,\text{sec}$

$\Rightarrow v_c(t) = v_c(0)\,e^{-t/\tau} = 12\,e^{-2t}\,V$

$\Rightarrow i_c(t) = C\dfrac{dv_c(t)}{dt} = -4e^{-2t}\,A$

By substitution thm :



From circuit (*) we know that

$i_1 = \frac{1}{2}i_s$ & $i_2 = \frac{3}{4}i_s$

$\Rightarrow v_x = 6(i_2 - i_1) = 6\left(\frac{3}{4}i_s - \frac{1}{2}i_s\right) = \frac{3}{2}i_s$

$\Rightarrow v_x(t) = \frac{3}{2}\{4e^{-2t}\} = \boxed{6e^{-2t}\,V}$