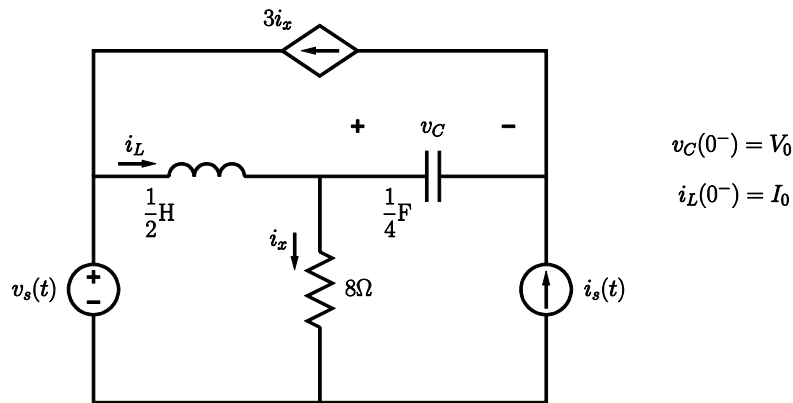


HOMEWORK VIII

Question 1 Consider the following circuit.

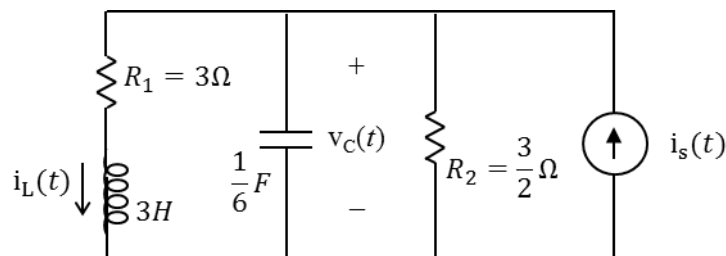


- Obtain the state equation. Determine the natural frequencies of the circuit and the natural frequencies observed by each state variable.
- Solve the state equation by Laplace transformation, and express $V_C(s)$ and $I_L(s)$ in terms of $V_s(s)$, $I_s(s)$, V_0 , and I_0 .
- Transform the circuit to the s -domain and obtain the node equation.
- Express the Laplace transforms of node voltages in terms of $V_s(s)$, $I_s(s)$, V_0 , and I_0 . Then express $V_C(s)$ and $I_L(s)$ in terms of $V_s(s)$, $I_s(s)$, V_0 , and I_0 .

For the remainder of the problem, assume $V_0 = 0$ and $I_0 = 0$.

- Let $i_s(t) = \delta(t)$ A and $v_s(t) = 0$. Using the initial and final value theorems find $v_C(0^+)$, $i_L(0^+)$, $v_C(+\infty)$, and $i_L(+\infty)$.
- Let $i_s(t) = u(t)$ A and $v_s(t) = 0$. Find $v_C(t)$ for $t \geq 0$.
- Let $v_s(t) = 5\cos(2t + 12^\circ)u(t)$ V and $i_s(t) = 0$. Find $i_L(t)$ for $t \geq 0$.

Question 2 Consider the following circuit.



- Obtain the state equation.
- Find the transfer function $H(s) = I_L(s)/I_s(s)$ and write the differential equation satisfied by $i_L(t)$.

- c) When $i_s(t) = 60 \cos(\omega_1 t)$ mA, we measure $i_L(t) = I_m \sin(\omega_1 t)$ mA in the steady state. Find ω_1 and I_m .
- d) Let $i_s(t) = 0$. Suppose that the ratio of the instantaneous powers absorbed by the resistors is constant: $\frac{p_1(t)}{p_2(t)} = K$. Find the possible values of K .

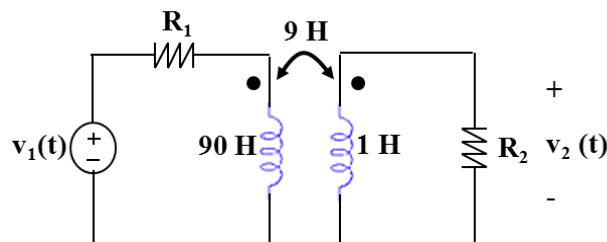
Answer b) $H(s) = \frac{2}{s^2 + 5s + 6}$. c) $\omega_1 = \sqrt{6}$ rad/sec, $I_m = 4\sqrt{6}$ A. d) $K = \left\{\frac{1}{2}, \frac{1}{8}\right\}$.

Question 3 The Laplace transform of the unit step response of a linear time invariant circuit is given as

$$V_L(s) = \frac{s^2 + 4}{s^3 + 6s^2 + 8s}.$$

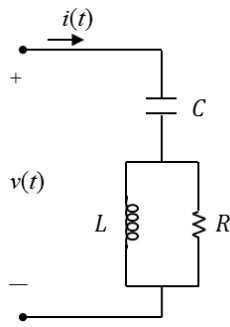
- a) Find the system function $H(s)$.
- b) Obtain the pole-zero plot.
- c) Find the impulse response.
- d) Sketch the magnitude and phase characteristics.
- e) Find the zero state response if the input is $v_i(t) = 2\delta(t) + 4u(t - 1)$ V.
- f) Find the steady state response if the input is $v_i(t) = 3\cos(4t - 8^\circ) - 7\cos(2t + 19^\circ)$ V.

Question 4 Consider the circuit below.



- a) Obtain the transfer function $H(s) = V_2(s)/V_1(s)$.
- b) Let $v_1(t) = 20\cos(\omega t)$ V and the circuit be in the sinusoidal steady state. The average power P delivered to R_2 is a function of ω , R_1 and R_2 .
Given R_1 and R_2 , find the value ω_m of ω (in terms of R_1 and R_2) so that P is maximized.
- c) Let $\omega = \omega_m$ and $R_1 = 1.8$ k Ω . Find the value of R_2 so that P is maximized. Determine P and interpret the result.

Question 5 Consider the one-port circuit given below.



$$L = 1/2 \text{ H}, \quad C = 1/8 \text{ F}, \quad R = 10 \, \Omega.$$

- Find the resonant frequency ω_r .
- Obtain the input admittance $Y(s)$, first, in terms of R , L and C ; then substitute the element values. Obtain the pole/zero plot.
- Find $Y(j\omega_r)$.
- Determine the quality factor of the circuit.
- Roughly sketch the magnitude and phase characteristics for $Y(j\omega)$.
- If the above one-port is approximated around ω_r by the series RLC one-port, what would be the value of the equivalent resistance R_{eq} ?
- Scale the circuit (i.e., find C) so that $L = 10 \text{ mH}$ and $R = 1 \text{ k}\Omega$. What is the resonant frequency of the scaled circuit?
- Reconsider the unscaled circuit. Let $i(t) = I_m \cos(\omega_r t)$, the one-port be in the sinusoidal steady-state, and the average stored energy in the capacitor be $E_c = 3/2 \text{ J}$. Find I_m , the port voltage $v(t)$, and the average power delivered to R .

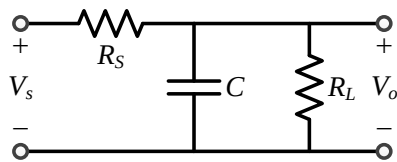
Answer a) $\omega_r = 4\sqrt{25/24} \text{ rad/sec}$ b) $Y(s) = \frac{1}{10} \frac{s(s+20)}{s^2 + 0.8s + 16}$ d) $Q = 5$ f) $R_{eq} = 0.4 \, \Omega$

g) $\omega_r^{after} = 20\sqrt{25/24} \text{ krad/sec}$

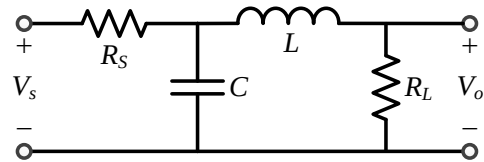
Question 6 Consider the transfer function:

$$H(s) = \frac{V_o(s)}{V_s(s)} = \frac{K}{s^2 + 4s + 16}.$$

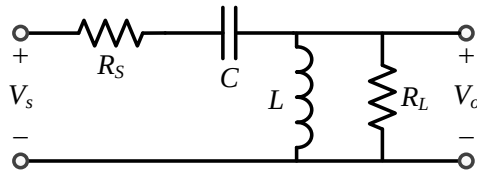
- Obtain the pole/zero plot.
- Sketch the magnitude and phase characteristics for $K = 16$.
- Which of the circuit topologies shown below can realize this transfer function? Explain your reasoning.



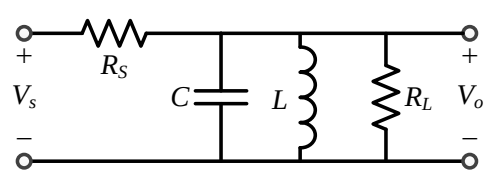
(a)



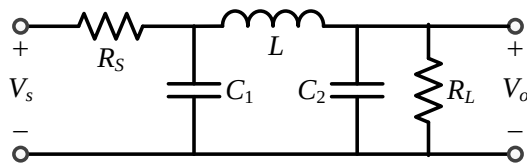
(b)



(c)



(d)



(e)

- d)** For $R_S = 1 \Omega$, for the topology you have stated in Part (c), find suitable element values that yield the desired transfer function. Also find K .
- e)** The circuit of Part (d) is so scaled that $R_S = 1 \text{ k}\Omega$ and $L = 1/9 \text{ H}$. Find all the element values and K . Sketch the magnitude and phase characteristics for the scaled circuit.

Question 7

- a)** The magnitude response function for an n^{th} order ($n = 1, 2, 3, \dots$) lowpass Butterworth filter is

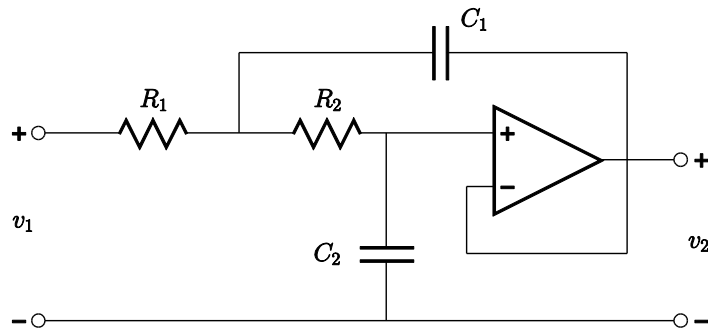
$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^{2n}}}.$$

The transfer function of a second order lowpass Butterworth filter is

$$H(s) = \frac{\omega_0^2}{s^2 + 2\alpha s + \omega_0^2}.$$

Express 2α in terms of ω_0 .

- b)** Consider the below circuit where the op-amp is ideal and operates in the linear region.



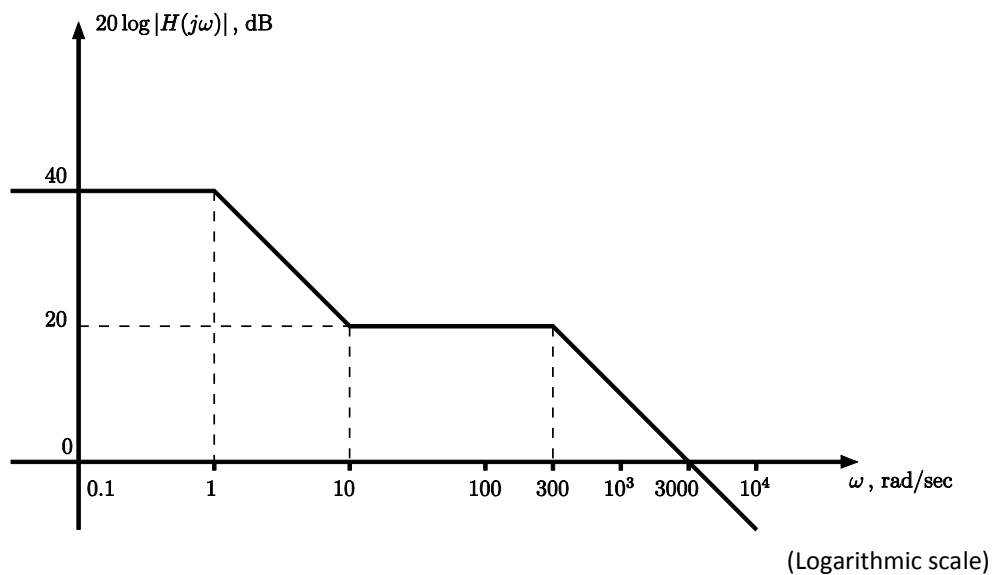
- i) Find the transfer function $H(s) = V_2(s)/V_1(s)$.
- ii) Design the circuit (i.e., determine R_1, R_2, C_1, C_2 values) so that it is a second order Butterworth filter with $\omega_0 = 1$ rad/sec. *Hint: Try $C_1 = 2C_2$.*
- iii) Scale the circuit so that $C_2 = 0.01 \mu\text{F}$ and the half-power bandwidth is 10 krad/sec.

Question 8 Consider the system function

$$H(s) = \frac{128(s^2 + 4)}{(s + 32)(s^2 + 8s + 64)}.$$

- a) Obtain the pole/zero plot.
- b) Obtain the magnitude and phase Bode plots.

Question 9 The straight line approximation for the magnitude Bode plot of a system function $H(s)$ is given below.



- a) Find $H(s)$.
- b) Draw the straight line approximation to the phase Bode plot of $H(s)$.