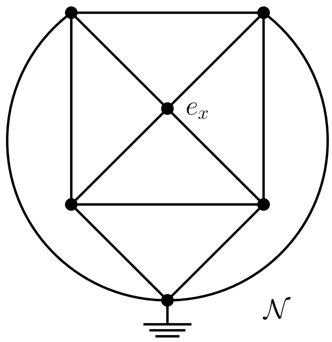
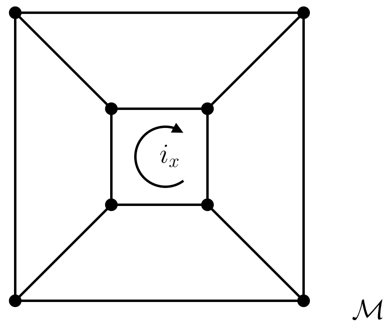
Despite its theoretical excellence the duality between mesh current and node voltage seems to suffer a practical imperfection: given a planar circuit, it is (we are made believe by many textbooks) not in general possible to directly measure the mesh currents, while the node voltages in the dual circuit can always be read by a voltmeter. For this reason mesh currents, unlike node voltages, are considered not to be physically real. They are instead treated as mere analytical tools with no projection onto reality and thus said to be *fictitious* or *imaginative*. Here we intend to take the opposite view.[1] That is, mesh currents are as physically real as node voltages; not a bit less, not a bit more. The mathematical symmetry between mesh current and node voltage hence beautifully extends to the physical world. We now proceed to illustrate this symmetry on a pair of dual circuits.

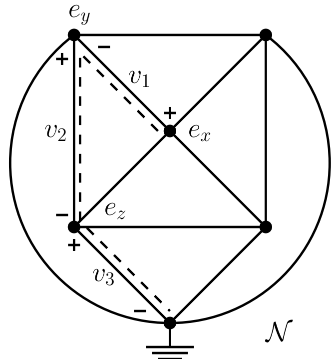
Let us study the below dual circuits $\mathcal{N}$ and $\mathcal{M}$, where the node voltage e_x of $\mathcal{N}$ and the mesh current i_x of $\mathcal{M}$ are dual variables. Let n_x denote the node that e_x is associated to. Likewise, m_x denotes the mesh that i_x is associated to. Now, for the circuit $\mathcal{N}$ it is clear how to measure e_x by a voltmeter. But how about the dual measurement for the circuit $\mathcal{M}$, i.e., how do we measure i_x? To answer this question let us invoke duality.

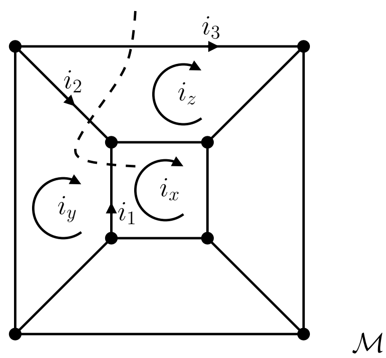




Consider the circuit $\mathcal{N}$. Note that "e_x equals the sum of branch voltages associated to any path that connects the node n_x to the ground node." The dual of this statement for $\mathcal{M}$ should read: "i_x equals the sum of branch currents associated to any cut that connects the mesh m_x to the outer mesh." For example, take the path (in $\mathcal{N}$) and its dual cut (in $\mathcal{M}$) shown below. For this particular path-cut pair our dual statements yield

$$\begin{aligned} e_x &= (e_x - e_y) + (e_y - e_z) + e_z \\ &= v_1 + v_2 + v_3\,, \end{aligned} \qquad\qquad \begin{aligned} i_x &= (i_x - i_y) + (i_y - i_z) + i_z \\ &= i_1 + i_2 + i_3\,. \end{aligned}$$

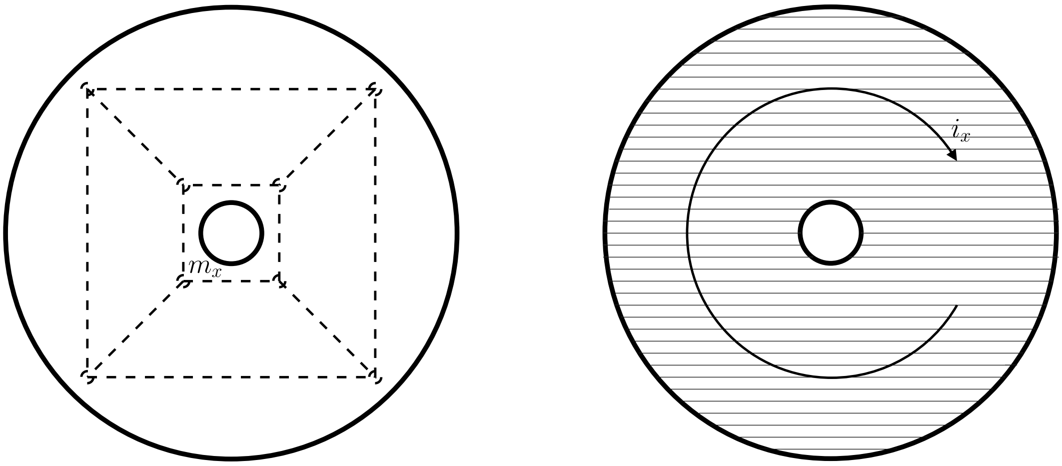




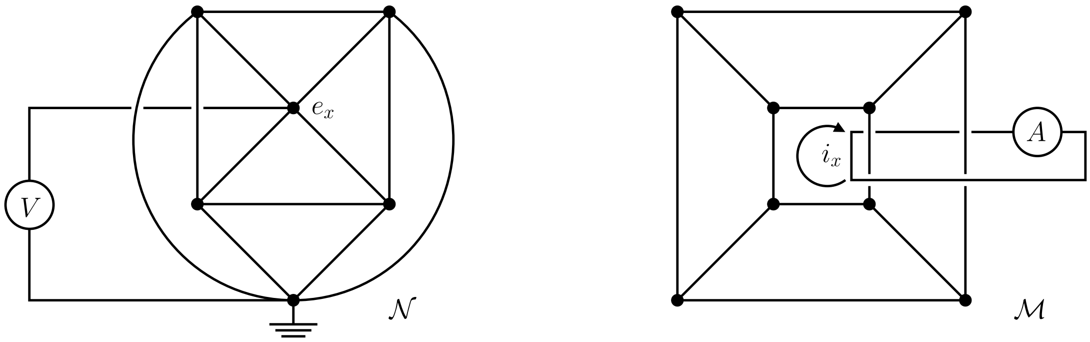
The fact that i_x is independent of the cut, provided that the cut connects m_x to the outer mesh, gives i_x at once a physical character. Namely, *the mesh current i_x is the (well-defined) current circulating in the ring (or "donut") where the inner circle of the ring is contained in the mesh m_x while the outer circle of*

[1] C.W. Cox. On the nature of mesh currents. *IEEE Transactions on Education*, 10(1):43-43, 1967.

the ring encapsulates the outer mesh; see the below figure.



Thanks to this simple observation the question on measuring i_x we asked earlier can now be rephrased: What instrument does one use to measure the current circulating within a "donut"? The answer is *clamp-on ammeter*. The below figure shows the dual measurements of e_x and i_x by a voltmeter and a clamp-on ammeter, respectively. This allows us to conclude that mesh currents are as measurable and hence as real as node voltages.



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Exercise. Show that the below measurements are dual.