On the Impact of Fast-Time and Slow-Time Preprocessing Operations on Adaptive Target Detectors

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Abstract-Conventional adaptive detectors assume independent and identically distributed (iid) secondary data vectors which is not contaminated with the target signal. Yet, the input to the adaptive detectors are produced by preprocessing the raw data in fast-time and slow-time dimensions, in general. This paper mainly aims to study the impact of fast-time matched filtering on the adaptive target detectors, namely Kelly's detector, adaptive matched filter (AMF) and adaptive coherence estimator (ACE). It is shown that the application of matched filtering prior to the adaptive detection violates the requirements of conventional adaptive detectors unless the range side-lobes of the radar pulse is zero at all lags. An alternative preprocessing method, based on an unitary transformation mapping, is suggested and it is shown the alternative approach exactly satisfies the requirements. Numerical comparisons are provided to examine the performance gain of the suggested approach in comparison with the conventional one, i.e. fast-time matched filtering.

Index Terms—Adaptive Detectors, Kelly's Detector, Adaptive Matched Filter, Adaptive Coherence Estimator.

I. INTRODUCTION

Adaptive detectors utilize secondary data vectors, in addition to the primary data vector representing the data for the cell-under-test (CUT), for the suppression of clutter component possibly prohibiting the detection of a target in CUT. The performance of adaptive detectors has been investigated from both practical and theoretical viewpoints for decades, [1], [2], [3]. Yet, due to the presence of several scenario specific factors (such as the spatial homogenity of texture parameter, spatial correlation of clutter across range cells, the Gaussianity assumption for the clutter, the size of secondary data, the mismatch of the steering vector etc.), it is difficult to present a clear taxonomy of detectors with respect to their performance in all scenarios of interest. In this study, we focus on another effect on the performance of adaptive radar detectors which is the fast-time operations, i.e. pulse matched filtering and its alternatives.

The work on adaptive detectors has been initiated with the work of Kelly where both primary and secondary data have been included in the hypothesis test, [4]. Different from earlier ad-hoc approaches, where the secondary data is used solely for the estimation of clutter statistics, Kelly's model enables the utilization of secondary data vectors along with the primary data in the detector design. The signal model of Kelly has been extended to cover subspace detectors [1], [5] and other detector families with the acronyms of AMF, ACE [6], [7].

We invite readers to examine recent monographs for further information on the extensions of Kelly's work, [1], [8].

The initial assumption in the development of adaptive detectors is the availability of signal-free snapshot vectors clutter vectors as the secondary data. Kelly explicitly states this fact in the problem formulation section of [4] as "The secondary data are assumed to be free of signal components, at least in the design of the algorithm, and any selection rules applied to make this assumption more plausible are ignored." It should also be remembered that the conventional processing chain for a radar system includes a pulse matched filter at the front end of the operations. Assuming that the signal is present at CUT, then several cells on both sides of the CUT are contaminated with the signal proportional to the pulse auto-correlation sequence (also known as range sidelobes), if the pulse matched filtering is applied prior to the adaptive detection. Typically, for high-resolution systems, the pulses of high bandwidth and high duration (high time-bandwidth product pulses) are utilized to establish high range resolution at a low probability of intercept. For such systems, declaring the several cells around the CUT as non-suitable cells (guard cells) for the secondary data, due to target signal contamination, results in the adoption of cells which are physically located at a large distance from the CUT cell. Increasing the physical separation between the primary cell and secondary data cells is not desired, since the homogenity assumption of the primary and secondary data cells is challenged more and more with the increased distance.

The main goal of this study is to study the impact of signal contamination due to the fast-time processing (pulse matched filtering) for different adaptive detection schemes. We present an alternative approach to the matched filtering operation that eliminates signal contamination in secondary data cells altogether and compare the performance of this approach with the conventional matched filtering based approach.

II. PROBLEM DEFINITION

With no loss of generality, we present the problem for a specific system set-up, which is a pulse Doppler radar system utilizing N pulses at a coherent processing interval. (The description given below also applies to space-time processing schemes where several receiving elements sample the time signal of interest synchronously.) It is assumed that the radar pulse has the length of K + 1 samples. For the sake of clarity,



Fig. 1. An illustration for the signal matrix S model for adaptive detectors

the pulse can be considered as a coded pulse with the code length of K + 1, where the sampling rate can be 1 sample per chip.

Figure 1 illustrates the signal matrix for the described scenario. The rows of this matrix indicates the returns collected due to each pulse transmission. The columns of the matrix are the collection of fast-time samples at a specific range. The hypothesis test for the detection problem can be described as follows. Our goal is to detect the presence or absence of a return due to a target located at the range shown as CUT in Figure 1. If the target is present, the CUT cell and K cells (shown with gray color in Figure 1)following the CUT cell contains signal energy due to the transmitted pulse. The samples of the radar pulse is denoted with b_k for $k = \{0, 1, \ldots, K\}$ in Figure 1.

Due to motion of the target during the coherent processing interval (CPI), there exists a pulse-to-pulse phase progression in slow-time, which is the Doppler effect. This phase progression is shown with $\mathbf{d}_{\phi} = \begin{bmatrix} 1 & e^{j\phi} & \dots & e^{j(N-1)\phi} \end{bmatrix}^T$ where $\phi = 2\pi f_d/\text{PRF}$ (f_d is the the Doppler frequency associated with range rate of the target. PRF is the pulse repetition frequency). The elements of \mathbf{d}_{ϕ} vector is denoted by d_n and d_n appears the common factor multiplying the return from the target for each pulse, i.e. slow-time sample.

With this signal model, the hypothesis test can be written as follows:

$$\begin{cases} H_0: \mathbf{r}_k = \mathbf{n}_k, & k = \{0, 1, \dots, K\} \\ H_1: \mathbf{r}_k = \rho \mathbf{d}_{\phi} b_k + \mathbf{n}_k, & k = \{0, 1, \dots, K\} \end{cases}$$
(1)

where k = 0 corresponds to the CUT cell in Figure 1 and K cells following the CUT are the cells that can possibly contain the scaled samples of the transmitted pulse.

The vectors \mathbf{n}_k for k={0,1,..., K} in (1) are independent and identically distributed (iid) circularly symmetric complex Gaussian vectors with the zero mean and covariance matrix **M**. The parameter ρ appearing in H_1 hypothesis is a non-random, unknown parameter in relation with the signal-to-noise-ratio (SNR). We assume that \mathbf{d}_{ϕ} and \mathbf{b}^H are known vectors. (In practice, the hypothesis test is repeated for several \mathbf{d}_{ϕ} vectors.)

By concatenating the data vectors into the columns of $N \times (K + 1)$ dimensional observation matrix $\mathbf{R} = [\mathbf{r}_0 \ \mathbf{r}_1 \ \dots \ \mathbf{r}_K]$ and defining $\mathbf{N} = [\mathbf{n}_0 \ \mathbf{n}_1 \ \dots \ \mathbf{n}_K]$, $\mathbf{b}^{H} = \begin{bmatrix} b_0 & b_1 & \dots & b_K \end{bmatrix}$; we can express the hypothesis test in (1) as

$$\begin{cases} H_0 : \mathbf{R} = \mathbf{N} \\ H_1 : \mathbf{R} = \rho \mathbf{d}_{\phi} \mathbf{b}^H + \mathbf{N} \end{cases}$$
(2)

The distribution for H_1 hypothesis can be written as

$$f_{\mathbf{R}}(\mathbf{R}; H_1) = \frac{\exp\left(-\operatorname{tr}\{\mathbf{M}^{-1}(\mathbf{R} - \mathbf{S})(\mathbf{R} - \mathbf{S})^H\}\right)}{\pi^{N(K+1)}|\mathbf{M}|^{K+1}} \qquad (3)$$

where $\mathbf{S} = \rho \mathbf{d}_{\phi} \mathbf{b}^{H}$ is the rank-1 signal matrix. The density for the H_0 hypothesis is identical in expression to (3) with the substitution of $\mathbf{S} = 0$.

Without any loss of generality, we assume that $\|\mathbf{b}\| = 1$ (unit energy radar pulse), since the factor ρ in (2) is an unknown non-random parameter which can be considered to absorb the value of $\|\mathbf{b}\|$.

The adaptive target detection problem is to construct the detector for the hypothesis test in (2) when both ρ and M are unknown. Kelly's approach is to derive the generalized likelihood ratio (GLRT) by jointly estimating unknown parameters and substituting the estimates into GLRT relation.

III. PREPROCESSING OPERATIONS BEFORE ADAPTIVE DETECTION

This section studies the preprocessing operations before the application of adaptive detectors. The preprocessing operations can be categorized into two as fast-time and slow-time operations. The fast-time processing is, typically, the matched filtering operation and applied to the rows of signal matrix shown in Figure 1. The slow-time processing (also called as Doppler processing) is a linear mapping (such as discrete Fourier transform, i.e. DFT) and applied on the columns of the signal matrix.

It is well known that for non-random parameter estimation under additive Gaussian noise (which is ρ for the problem of interest), the matched filtering operation (in general the whitened matched filtering operation) yields the sufficient statistics, provided that the covariance matrix of the noise (**M** matrix for the problem of interest) is known exactly, [8, p.24]. For adaptive detectors, the covariance matrix **M** is unknown and its estimate is generated at an intermediate stage of the GLRT based detection approach. Hence, the suitability of the matched filtering operation prior to the adaptive detection is not immediately clear, given that **M** matrix is an unknown of the problem.

Similarly, the application of a linear mapping, say \mathbf{U}_D^H , on the columns of signal matrix $\mathbf{S} = \mathbf{d}_{\phi} \mathbf{b}^H$ is equivalent to converting the Doppler steering vector \mathbf{d}_{ϕ} to $\mathbf{U}_D^H \mathbf{d}_{\phi}$. This operation is, typically, implemented to reduce the dimension of the $N \times 1$ snapshot vectors forming the columns of the signal matrix to a much smaller dimensional vectors, say Ddimensional vectors where $D \ll N$. The goal of dimension reduction operation is to improve the estimation accuracy of the unknown covariance matrix \mathbf{M} , [9], and also reduce the computational load at the receiver.

This section aims to examine both fast-time and slow-time operations that may take place before the application adaptive detectors. From an abstract viewpoint, the presence or absence of the target signal, i.e. the signal matrix, in the observation matrix \mathbf{R} is investigated. The fast-time and slow-time linear operations corresponds to a right and left multiplications by matrices say \mathbf{Q} and \mathbf{U}_D^H , respectively. Stated differently, preprocessing operations convert the observation matrix \mathbf{R} to $\mathbf{U}_D^H \mathbf{R} \mathbf{Q}$. It should be immediately clear from this expression that that the order of fast-time and slow-time operations is indeed inconsequential.

A. Fast-time preprocessing

Conventional Approach: Matched Filtering The application of matched filtering operation on data matrix \mathbf{R} can be expressed as the multiplication of \mathbf{R} with a Toeplitz matrix whose first column is the vector \mathbf{b} , i.e.

$$\mathbf{Q_b} = \begin{bmatrix} b_0^* & 0 & \dots & 0 \\ b_1^* & b_0^* & \dots & 0 \\ b_2^* & b_1^* & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ b_K^* & b_{K-1}^* & \dots & 0 \\ 0 & b_K^* & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & b_K^* \end{bmatrix}_{(2K+1) \times (K+1)}$$
(4)

The matched filtering operation on **R** matrix can be expressed as $\widetilde{\mathbf{R}} = \mathbf{R}\mathbf{Q}_{\mathbf{b}}^{-1}$.

After matched filtering, the hypothesis test becomes

$$\begin{cases} H_0 : \widetilde{\mathbf{R}} = \widetilde{\mathbf{N}} \\ H_1 : \widetilde{\mathbf{R}} = \rho \mathbf{d}_{\phi} \mathbf{r}_{\mathbf{b}}^H + \widetilde{\mathbf{N}} \end{cases}$$
(5)

where $\mathbf{r}_{\mathbf{b}}^{H}$ vector is the auto-correlation vector for the radar pulse.

If the auto-correlation sequence of the pulse b_k is close to ideal, that is $r_b[k] \approx \delta[k]$, then the test in (5) can be said to approximately satisfy the conditions of the Kelly's test. It should be remembered that Kelly's test requires that the columns of \tilde{N} matrix to be iid Gaussian vectors with common variance matrix of M and also assumes that secondary data cells are free of signal contamination. It can be checked that the condition of ideal auto-correlation for the radar pulse, i.e. zero range side-lobes, is required for both requirements to hold exactly.

With the assumption of $r_b[k] = \delta[k]$, Kelly's test after matched filtering operation becomes

$$t_{\text{Kelly-mf}} = \frac{|\widetilde{\mathbf{r}}_0^H \mathbf{S}^{-1} \mathbf{d}_{\phi}|^2}{\mathbf{d}_{\phi}^H \mathbf{S}^{-1} \mathbf{d}_{\phi} (1 + \widetilde{\mathbf{r}}_0^H \mathbf{S}^{-1} \widetilde{\mathbf{r}}_0)} \overset{H_1}{\underset{H_0}{\gtrless}} \gamma \tag{6}$$

where $\mathbf{S} = \sum_{k=1}^{K} \widetilde{\mathbf{r}}_k \widetilde{\mathbf{r}}_k^H$. $\widetilde{\mathbf{r}}_k$ are the columns of $\widetilde{\mathbf{R}}$ matrix where k = 0 corresponds to the CUT cell and others correspond to the secondary data cells. Full information on the derivation of Kelly's test is available at [1].

Second Approach: Unitary Transformation As noted by Steinhardt in [10, p.140], it is possible to eliminate the signal contamination on the secondary data with the application of

an unitary transformation. To this aim, we define a $K \times K$ $\mathbf{Q}_{\mathbf{u}}$ matrix whose first column is the unit norm **b** vector and remaining columns are the vectors orthogonal to **b**, that is $\mathbf{Q}_{\mathbf{u}} = \begin{bmatrix} \mathbf{b} & \mathbf{b}_{1}^{\perp} & \dots & \mathbf{b}_{K-1}^{\perp} \end{bmatrix}$. Vectors orthogonal to **b** are denoted as \mathbf{b}_{k}^{\perp} . Upon the application of $\mathbf{Q}_{\mathbf{u}}$ matrix to the observation matrix **R** from right, i.e. $\overline{\mathbf{R}} = \mathbf{R}\mathbf{Q}_{\mathbf{u}}$, we arrive at the modified hypothesis test

$$\begin{cases} H_0 : \overline{\mathbf{R}} = \overline{\mathbf{N}} \\ H_1 : \overline{\mathbf{R}} = \rho \mathbf{d}_{\phi} \mathbf{e}_1^H + \overline{\mathbf{N}} \end{cases}$$
(7)

where $\overline{\mathbf{N}}$ is the random matrix representing collection of noise vectors after the application of $\mathbf{Q}_{\mathbf{u}}$ from right. Since, the columns of $\widetilde{\mathbf{N}}$ matrix are iid distributed Gaussian vectors, the application of a unitary matrix from right does not change the statistics of noise matrix. Hence, the only effect of right multiplication by $\mathbf{Q}_{\mathbf{u}}$ is on the signal matrix. After this operation, the signal matrix becomes $\mathbf{d}_{\phi}\mathbf{e}_{1}^{H}$ where $\mathbf{e}_{1}^{H} = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}$ is the first canonical basis vector. Hence, with the application of unitary transformation, different from matched filtering case, the assumptions for the Kelly's test are exactly satisfied.

In order to use the results of the conventional Kelly test, we express the CUT vector after right multiplication by $\mathbf{Q}_{\mathbf{u}}$ as $\mathbf{\bar{r}}_0 = \mathbf{R}\mathbf{b}$. The **S** matrix after right multiplication operation $\mathbf{S}_{\mathbf{Q}_{\mathbf{u}}} = \sum_{k=1}^{K} \mathbf{\bar{r}}_k \mathbf{\bar{r}}_k^H$ where $\mathbf{\bar{r}}_k$ is the k'th column of product $\mathbf{\bar{R}} = \mathbf{R}\mathbf{Q}$. Adding and subtracting outer product corresponding to the CUT cell to $\mathbf{S}_{\mathbf{Q}_{\mathbf{u}}}$, we get

$$\mathbf{S}_{\mathbf{Q}_{\mathbf{u}}} = \sum_{k=0}^{K} \overline{\mathbf{r}}_{k} \overline{\mathbf{r}}_{k}^{H} - \overline{\mathbf{r}}_{0} \overline{\mathbf{r}}_{0}^{H}$$

$$= \overline{\mathbf{R}} \overline{\mathbf{R}}^{H} - (\mathbf{R}\mathbf{b})(\mathbf{R}\mathbf{b})^{H}$$

$$= \mathbf{R} \underbrace{\mathbf{Q}_{\mathbf{u}}}_{\mathbf{I}} \mathbf{Q}_{\mathbf{u}}^{H} \mathbf{R} - \mathbf{R}\mathbf{b}\mathbf{b}^{H} \mathbf{R}^{H}$$

$$= \mathbf{R} (\mathbf{I} - \mathbf{b}\mathbf{b}^{H}) \mathbf{R}^{H}. \qquad (8)$$

It can be noted that $(\mathbf{I} - \mathbf{b}\mathbf{b}^H)$ is the projection matrix to the orthogonal complement of the subspace spanned by vector \mathbf{b} . It can be checked that the projection operation eliminates the signal contamination in the secondary cells altogether.

With these modifications, Kelly's test becomes

$$t_{\text{Kelly-u}} = \frac{|\overline{\mathbf{r}}_0^H \mathbf{S}_{\mathbf{Q}_u}^{-1} \mathbf{d}_{\phi}|^2}{\mathbf{d}_{\phi}^H \mathbf{S}_{\mathbf{Q}_u}^{-1} \mathbf{d}_{\phi} (1 + \overline{\mathbf{r}}_0^H \mathbf{S}_{\mathbf{Q}_u}^{-1} \overline{\mathbf{r}}_0)} \overset{H_1}{\underset{H_0}{\gtrless}} \gamma.$$
(9)

In addition to Kelly's test, the adaptive matched filter (AMF) test, given in [6], can be also he expressed after the fast-time operations of matched filtering or unitary transformation as follows:

$$t_{\text{AMF-mf}} = \frac{|\mathbf{\tilde{r}}_{0}^{H}\mathbf{S}^{-1}\mathbf{d}_{\phi}|^{2}}{\mathbf{d}_{\phi}^{H}\mathbf{S}^{-1}\mathbf{d}_{\phi}} \overset{H_{1}}{\underset{H_{0}}{\gtrsim}} \gamma,$$

$$t_{\text{AMF-u}} = \frac{|\mathbf{\bar{r}}_{0}^{H}\mathbf{S}_{\mathbf{Q}_{u}}^{-1}\mathbf{d}_{\phi}|^{2}}{\mathbf{d}_{\phi}^{H}\mathbf{S}_{\mathbf{Q}_{u}}^{-1}\mathbf{d}_{\phi}} \overset{H_{1}}{\underset{H_{0}}{\gtrsim}} \gamma.$$
(10)

Similarly, the adaptive coherence test, given in [7], can be

¹We assume that in the construction of **R** matrix, the amount of secondary cells shown in Figure 1 is doubled to 2K vectors and **S** matrix is zeropadded with K column vector of zeros to a dimension of $N \times (2K + 1)$ so that the matched filtering operation $\mathbf{RQ}_{\mathbf{b}}$ can be expressed with matrices of compatible dimensions. We do not introduce more notation into the problem in order to simply the delivery of our main message.

expressed after fast-time operations as:

$$t_{\text{ACE-mf}} = \frac{|\mathbf{\widetilde{r}}_{0}^{H}\mathbf{S}^{-1}\mathbf{d}_{\phi}|^{2}}{\left(\mathbf{d}_{\phi}^{H}\mathbf{S}^{-1}\mathbf{d}_{\phi}\right)\left(\mathbf{\widetilde{r}}_{0}^{H}\mathbf{S}^{-1}\mathbf{\widetilde{r}}_{0}\right)} \overset{H_{1}}{\underset{H_{0}}{\gtrless}} \gamma,$$

$$t_{\text{ACE-u}} = \frac{|\mathbf{\overline{r}}_{0}^{H}\mathbf{S}_{\mathbf{Q}_{u}}^{-1}\mathbf{d}_{\phi}|^{2}}{\left(\mathbf{d}_{\phi}^{H}\mathbf{S}_{\mathbf{Q}_{u}}^{-1}\mathbf{d}_{\phi}\right)\left(\mathbf{\overline{r}}_{0}^{H}\mathbf{S}_{\mathbf{Q}_{u}}^{-1}\mathbf{\overline{r}}_{0}\right)} \overset{H_{1}}{\underset{H_{0}}{\gtrless}} \gamma. \quad (11)$$

B. Slow-time preprocessing

Slow-time processing corresponds to the left multiplication of the observation matrix \mathbf{R} with $D \times N$ dimensional \mathbf{U}_D^H matrix, as noted before. After this operation, the hypothesis test becomes

$$\begin{cases}
H_0 : \widehat{\mathbf{R}} = \widehat{\mathbf{N}} \\
H_1 : \widehat{\mathbf{R}} = \rho \widehat{\mathbf{d}}_{\phi} \mathbf{b}^H + \widehat{\mathbf{N}}
\end{cases}$$
(12)

where $\widehat{\mathbf{d}}_{\phi} = \mathbf{U}_D^H \mathbf{d}_{\phi}$ is the $D \times 1$ dimensional Doppler steering vector. The columns of the matrix $\widehat{\mathbf{N}}$ are iid distributed Gaussian vectors with $D \times D$ covariance matrix $\widehat{\mathbf{M}} = \mathbf{U}_D^H \mathbf{M} \mathbf{U}_D$. Hence, the left multiplication by \mathbf{U}_D^H matrix does not result in any violations on the requirements for the Kelly's test.

If U_D is selected as *D* columns of a $N \times N$ DFT matrix, the resultant processing is called post-doppler processing. Post-Doppler processing is one of many possibilities for dimension reduction via slow-time operations. In [11], the optimal dimension reduction operation, with respect to several criteria, has been found as the generalized eigenspace of signal and noise covariance matrices. Unfortunately, this result is not immediately applicable to adaptive detection problem, since the noise covariance matrix is an unknown of the problem.

Slow-time operations can be implemented before or after fast-time operations with no harm to the performance. If slowtime operations are implemented before fast-time operations, the size observation matrix reduces to $D \times K$ ($\hat{\mathbf{R}} = \mathbf{U}_D^H \mathbf{R}$) and the doppler steering vector in reduced dimensions becomes $\hat{\mathbf{d}}_{\phi} = \mathbf{U}_D^H \mathbf{d}_{\phi}$. For the second stage of preprocessing (fasttime), we only need to replace \mathbf{R} matrix in the expressions given earlier with $\hat{\mathbf{R}}$ and \mathbf{d}_{ϕ} vector with $\hat{\mathbf{d}}_{\phi}$.

IV. NUMERICAL RESULTS

We present a set of numerical results to examine the effects of fast-time processing in adaptive radar detection. We compare the performance of three adaptive detectors, namely, Kelly, AMF and ACE, and the conventional DFT based detector (without additional clutter suppression) with that of the genie aided Max-SINR filter, where the clutter covariance matrix is assumed to be exactly known (which is impossible with the use of finite dimensional secondary data). The performances of all detectors are obtained for the case of Doppler mismatch, i.e. the target can be present at any fractional Doppler bin, but the Doppler steering vectors are constructed to direct only at Doppler frequencies $\left\{\frac{k}{N}\right\}_{k=0}^{N-1}$ (normalized with PRF). That is to say, the detectors assume that the target is present only at the center of each Doppler bin, which is the source of Doppler mismatch.

In case of mismatch, we know that the target contamination in the secondary range cells may affect the performance of adaptive detectors, since the detectors after fast-time matched filtering (MF) considers the range sidelobes of the target as



Fig. 2. P_d versus normalized target Doppler frequency for N = 16, K = 16, and 4 range cells are used at SNR=20 dB

interference, therefore it suppresses the target signal in the CUT cell as well. All these detectors are forced to have the same probability of false alarm (P_{FA}), and their detection performances are compared for various scenarios. Common Parameters for the scenarios:

- Clutter is assumed to have Gaussian type power spectral density. Thus, the (m, n)th entry of the covariance matrix M is given by ρ^{(m-n)²}, and ρ = 0.999 is taken.
- Clutter-to-noise ratio (CNR)= 40 dB (for secondary data) with CUT/secondary cell texture parameter: 0 dB (homogeneous clutter)
- N = 16 slow-time pulses
- P3 code of length 16 is used (K = 16)
- $P_{FA} = 10^{-3}$
- Diagonal loading $(\lambda = 3N_0)$ is utilized to alleviate the effect of contamination, and increase the convergence rate when smaller number of secondary cells are used.
- Doppler subspace dimension (in post-Doppler processing) D = 3
- Secondary data: Total 4 range cells next to the CUT are used to estimate M.
- No fractional range offset is assumed.
- Single Swerling-1 target
- No jammer

In Fig. 2 and 3, the performances in terms of the probability of detection (P_d) of different adaptive detectors after conventional fast-time MF are provided in case of Doppler mismatch for signal-to-noise ratio (SNR) 20 and 50 dB. As the SNR increases, the effect of contamination becomes more severe, thus it leads to significant performance degradation in case of mismatch. Among other adaptive techniques, AMF is more robust to Doppler mismatch, whereas ACE has much higher sensitivity in case of contamination.

In order to show the effect of contamination more explicitly, in Fig. 4, the performances are demonstrated for different SNR values when the target is located at a normalized Doppler $\frac{3.1}{16}$. The performance degradation is much more apparent for Kelly and ACE in high SNR values.



Fig. 3. P_d versus normalized target Doppler frequency for N = 16, K = 16, and 4 range cells are used at SNR=50 dB



Fig. 4. P_d versus SNR for N = 16, K = 16, and 4 range cells are used, target located at normalized Doppler $\frac{3.1}{16}$



Fig. 5. P_d versus normalized target Doppler frequency for N = 16, K = 16, and 4 range cells are used at SNR=20 dB



Fig. 6. P_d versus normalized target Doppler frequency for N = 16, K = 16, and 4 range cells are used at SNR=50 dB

As an alternative to conventional fast-time MF, Unitary Transformation (explained in Section III) can be applied to get the target free secondary data without changing the correlation structure of the clutter in slow- and fast-time. In that case, the adaptive detectors outperforms the classical ones with MF by taking into account the unavoidable range sidelobes of the target, hence it estimates the clutter covariance better than the classical ones.

The advantages of this pre-processing is demonstrated in Fig. 5, 6 and 7, where there seems no dramatic performance degradation due to Doppler mismatch. This is attained without any increase in computational complexity. Therefore, we can conclude that the robustness of the adaptive detectors can be improved significantly in case of Doppler mismatch with the help of fast-time preprocessing introduced (especially for Kelly and AMF). Moreover, conventional DFT based detector performance is far from the ideal case, as expected. The adaptive detectors have better performances compared to the DFT

detector, as they are designed to learn the clutter covariance matrix from the secondary data. Adaptive detectors have some SNR loss in comparison to the genie-aided Max-SINR detector due to imperfect estimation of clutter covariance matrix.

V. CONCLUSIONS

The impact of signal contamination in the secondary range cells due to the fast-time processing is investigated for various adaptive detectors, namely, Kelly, AMF and ACE in case of target Doppler mismatch. An alternative approach to the fasttime matched filtering is introduced to eliminate the signal contamination (which is detrimental for adaptive operation in case of model mismatch) in the secondary data so that the robustness of the well-known adaptive detectors is enhanced considerably.

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Fig. 7. P_d versus SNR for N = 16, K = 16, and 4 range cells are used, target located at normalized Doppler $\frac{3.1}{16}$

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