variables, and we represent the corresponding s-domain variables with uppercase letters. Thus

$$\mathscr{L}\{v\} = V \quad \text{or} \quad v = \mathscr{L}^{-1}\{V\},$$

$$\mathscr{L}\{i\} = I \quad \text{or} \quad i = \mathscr{L}^{-1}\{I\},$$

$$\mathscr{L}\{f\} = F \quad \text{or} \quad f = \mathscr{L}^{-1}\{F\},$$

and so on.

NOTE: Assess your understanding of this material by trying Chapter Problem 12.26.

12.7 Inverse Transforms

The expression for $V(s)$ in Eq. 12.40 is a **rational** function of s; that is, one that can be expressed in the form of a ratio of two polynomials in s such that no nonintegral powers of s appear in the polynomials. In fact, for linear, lumped-parameter circuits whose component values are constant, the s-domain expressions for the unknown voltages and currents are always rational functions of s. (You may verify this observation by working Problems 12.28–12.31.) If we can inverse-transform rational functions of s, we can solve for the time-domain expressions for the voltages and currents. The purpose of this section is to present a straight-forward and systematic technique for finding the inverse transform of a rational function.

In general, we need to find the inverse transform of a function that has the form

$$F(s) = \frac{N(s)}{D(s)} = \frac{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}. \qquad (12.42)$$

The coefficients a and b are real constants, and the exponents m and n are positive integers. The ratio $N(s)/D(s)$ is called a **proper rational function** if $m > n$, and an **improper rational function** if $m \leq n$. Only a proper rational function can be expanded as a sum of partial fractions. This restriction poses no problem, as we show at the end of this section.

Partial Fraction Expansion: Proper Rational Functions

A proper rational function is expanded into a sum of partial fractions by writing a term or a series of terms for each root of $D(s)$. Thus $D(s)$ must be in factored form before we can make a partial fraction expansion. For each distinct root of $D(s)$, a single term appears in the sum of partial fractions. For each multiple root of $D(s)$ of multiplicity r, the expansion contains r terms. For example, in the rational function

$$\frac{s+6}{s(s+3)(s+1)^2},$$

the denominator has four roots. Two of these roots are distinct—namely, at $s = 0$ and $s = -3$. A multiple root of multiplicity 2 occurs at $s = -1$. Thus the partial fraction expansion of this function takes the form

$$\frac{s+6}{s(s+3)(s+1)^2} \equiv \frac{K_1}{s} + \frac{K_2}{s+3} + \frac{K_3}{(s+1)^2} + \frac{K_4}{s+1}. \qquad (12.43)$$

The key to the partial fraction technique for finding inverse transforms lies in recognizing the $f(t)$ corresponding to each term in the sum of partial fractions. From Table 12.1 you should be able to verify that

$$\mathcal{L}^{-1}\left\{\frac{s+6}{s(s+3)(s+1)^2}\right\}$$
$$= (K_1 + K_2e^{-3t} + K_3te^{-t} + K_4e^{-t})u(t). \tag{12.44}$$

All that remains is to establish a technique for determining the coefficients ($K_1, K_2, K_3, \ldots$) generated by making a partial fraction expansion. There are four general forms this problem can take. Specifically, the roots of $D(s)$ are either (1) real and distinct; (2) complex and distinct; (3) real and repeated; or (4) complex and repeated. Before we consider each situation in turn, a few general comments are in order.

We used the identity sign $\equiv$ in Eq. 12.43 to emphasize that expanding a rational function into a sum of partial fractions establishes an identical equation. Thus both sides of the equation must be the same for all values of the variable s. Also, the identity relationship must hold when both sides are subjected to the same mathematical operation. These characteristics are pertinent to determining the coefficients, as we will see.

Be sure to verify that the rational function is proper. This check is important because nothing in the procedure for finding the various Ks will alert you to nonsense results if the rational function is improper. We present a procedure for checking the Ks, but you can avoid wasted effort by forming the habit of asking yourself, "Is $F(s)$ a proper rational function?"

Partial Fraction Expansion: Distinct Real Roots of $D(s)$

We first consider determining the coefficients in a partial fraction expansion when all the roots of $D(s)$ are real and distinct. To find a K associated with a term that arises because of a distinct root of $D(s)$, we multiply both sides of the identity by a factor equal to the denominator beneath the desired K. Then when we evaluate both sides of the identity at the root corresponding to the multiplying factor, the right-hand side is always the desired K, and the left-hand side is always its numerical value. For example,

$$F(s) = \frac{96(s+5)(s+12)}{s(s+8)(s+6)} \equiv \frac{K_1}{s} + \frac{K_2}{s+8} + \frac{K_3}{s+6}. \tag{12.45}$$

To find the value of K_1, we multiply both sides by s and then evaluate both sides at $s = 0$:

$$\left.\frac{96(s+5)(s+12)}{(s+8)(s+6)}\right|_{s=0} \equiv K_1 + \left.\frac{K_2 s}{s+8}\right|_{s=0} + \left.\frac{K_3 s}{s+6}\right|_{s=0},$$

or

$$\frac{96(5)(12)}{8(6)} \equiv K_1 = 120. \tag{12.46}$$

To find the value of K_2, we multiply both sides by $s + 8$ and then evaluate both sides at $s = -8$:

$$\left.\frac{96(s+5)(s+12)}{s(s+6)}\right|_{s=-8}$$
$$\equiv \left.\frac{K_1(s+8)}{s}\right|_{s=-8} + K_2 + \left.\frac{K_3(s+8)}{(s+6)}\right|_{s=-8},$$

or

$$\frac{96(-3)(4)}{(-8)(-2)} = K_2 = -72. \tag{12.47}$$

Then K_3 is

$$\left.\frac{96(s+5)(s+12)}{s(s+8)}\right|_{s=-6} = K_3 = 48. \tag{12.48}$$

From Eq. 12.45 and the K values obtained,

$$\frac{96(s+5)(s+12)}{s(s+8)(s+6)} \equiv \frac{120}{s} + \frac{48}{s+6} - \frac{72}{s+8}. \tag{12.49}$$

At this point, testing the result to protect against computational errors is a good idea. As we already mentioned, a partial fraction expansion creates an identity; thus both sides of Eq. 12.49 must be the same for all s values. The choice of test values is completely open; hence we choose values that are easy to verify. For example, in Eq. 12.49, testing at either -5 or -12 is attractive because in both cases the left-hand side reduces to zero. Choosing -5 yields

$$\frac{120}{-5} + \frac{48}{1} - \frac{72}{3} = -24 + 48 - 24 = 0,$$

whereas testing -12 gives

$$\frac{120}{-12} + \frac{48}{-6} - \frac{72}{-4} = -10 - 8 + 18 = 0.$$

Now confident that the numerical values of the various Ks are correct, we proceed to find the inverse transform:

$$\mathcal{L}^{-1}\left\{\frac{96(s+5)(s+12)}{s(s+8)(s+6)}\right\} = (120 + 48e^{-6t} - 72e^{-8t})u(t). \tag{12.50}$$

✓ASSESSMENT PROBLEMS

Objective 2—Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

12.3 Find $f(t)$ if

$$F(s) = \frac{6s^2 + 26s + 26}{(s+1)(s+2)(s+3)}.$$

Answer: $f(t) = (3e^{-t} + 2e^{-2t} + e^{-3t})u(t)$.

12.4 Find $f(t)$ if

$$F(s) = \frac{7s^2 + 63s + 134}{(s+3)(s+4)(s+5)}.$$

Answer: $f(t) = (4e^{-3t} + 6e^{-4t} - 3e^{-5t})u(t)$.

NOTE: Also try Chapter Problems 12.40(a) and (b).

Partial Fraction Expansion: Distinct Complex Roots of $D(s)$

The only difference between finding the coefficients associated with distinct complex roots and finding those associated with distinct real roots is that the algebra in the former involves complex numbers. We illustrate by expanding the rational function:

$$F(s) = \frac{100(s + 3)}{(s + 6)(s^2 + 6s + 25)}. \tag{12.51}$$

We begin by noting that $F(s)$ is a proper rational function. Next we must find the roots of the quadratic term $s^2 + 6s + 25$:

$$s^2 + 6s + 25 = (s + 3 - j4)(s + 3 + j4). \tag{12.52}$$

With the denominator in factored form, we proceed as before:

$$\frac{100(s + 3)}{(s + 6)(s^2 + 6s + 25)} \equiv$$

$$\frac{K_1}{s + 6} + \frac{K_2}{s + 3 - j4} + \frac{K_3}{s + 3 + j4}. \tag{12.53}$$

To find K_1, K_2, and K_3, we use the same process as before:

$$K_1 = \left.\frac{100(s + 3)}{s^2 + 6s + 25}\right|_{s=-6} = \frac{100(-3)}{25} = -12, \tag{12.54}$$

$$K_2 = \left.\frac{100(s + 3)}{(s + 6)(s + 3 + j4)}\right|_{s=-3+j4} = \frac{100(j4)}{(3 + j4)(j8)}$$

$$= 6 - j8 = 10e^{-j53.13^\circ}, \tag{12.55}$$

$$K_3 = \left.\frac{100(s + 3)}{(s + 6)(s + 3 - j4)}\right|_{s=-3-j4} = \frac{100(-j4)}{(3 - j4)(-j8)}$$

$$= 6 + j8 = 10e^{j53.13^\circ}. \tag{12.56}$$

Then

$$\frac{100(s + 3)}{(s + 6)(s^2 + 6s + 25)} = \frac{-12}{s + 6} + \frac{10\angle -53.13^\circ}{s + 3 - j4}$$

$$+ \frac{10\angle 53.13^\circ}{s + 3 + j4}. \tag{12.57}$$

Again, we need to make some observations. First, in physically realizable circuits, complex roots always appear in conjugate pairs. Second, the coefficients associated with these conjugate pairs are themselves conjugates. Note, for example, that K_3 (Eq. 12.56) is the conjugate of K_2

(Eq. 12.55). Thus for complex conjugate roots, you actually need to calculate only half the coefficients.

Before inverse-transforming Eq. 12.57, we check the partial fraction expansion numerically. Testing at −3 is attractive because the left-hand side reduces to zero at this value:

$$F(s) = \frac{-12}{3} + \frac{10 \angle -53.13^\circ}{-j4} + \frac{10 \angle 53.13^\circ}{j4}$$

$$= -4 + 2.5 \angle 36.87^\circ + 2.5 \angle -36.87^\circ$$

$$= -4 + 2.0 + j1.5 + 2.0 - j1.5 = 0.$$

We now proceed to inverse-transform Eq. 12.57:

$$\mathscr{L}^{-1}\left\{\frac{100(s+3)}{(s+6)(s^2+6s+25)}\right\} = (-12e^{-6t} + 10e^{-j53.13^\circ}e^{-(3-j4)t}$$

$$+ 10e^{j53.13^\circ}e^{-(3+j4)t})u(t). \quad (12.58)$$

In general, having the function in the time domain contain imaginary components is undesirable. Fortunately, because the terms involving imaginary components always come in conjugate pairs, we can eliminate the imaginary components simply by adding the pairs:

$$10e^{-j53.13^\circ}e^{-(3-j4)t} + 10e^{j53.13^\circ}e^{-(3+j4)t}$$

$$= 10e^{-3t}\left(e^{j(4t-53.13^\circ)} + e^{-j(4t-53.13^\circ)}\right)$$

$$= 20e^{-3t}\cos(4t - 53.13^\circ), \quad (12.59)$$

which enables us to simplify Eq. 12.58:

$$\mathscr{L}^{-1}\left\{\frac{100(s+3)}{(s+6)(s^2+6s+25)}\right\}$$

$$= [-12e^{-6t} + 20e^{-3t}\cos(4t - 53.13^\circ)]u(t). \quad (12.60)$$

Because distinct complex roots appear frequently in lumped-parameter linear circuit analysis, we need to summarize these results with a new transform pair. Whenever $D(s)$ contains distinct complex roots—that is, factors of the form $(s + \alpha - j\beta)(s + \alpha + j\beta)$—a pair of terms of the form

$$\frac{K}{s + \alpha - j\beta} + \frac{K^*}{s + \alpha + j\beta} \quad (12.61)$$

appears in the partial fraction expansion, where the partial fraction coefficient is, in general, a complex number. In polar form,

$$K = |K|e^{j\theta} = |K| \angle \theta^\circ, \quad (12.62)$$

where $|K|$ denotes the magnitude of the complex coefficient. Then

$$K^* = |K|e^{-j\theta} = |K|\angle{-\theta^\circ}. \tag{12.63}$$

The complex conjugate pair in Eq. 12.61 always inverse-transforms as

$$\mathcal{L}^{-1}\left\{\frac{K}{s+\alpha-j\beta}+\frac{K^*}{s+\alpha+j\beta}\right\}$$
$$= 2|K|e^{-\alpha t}\cos(\beta t+\theta). \tag{12.64}$$

In applying Eq. 12.64 it is important to note that K is defined as the coefficient associated with the denominator term $s + \alpha - j\beta$, and K^* is defined as the coefficient associated with the denominator $s + \alpha + j\beta$.

✓ASSESSMENT PROBLEM

Objective 2—Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

12.5 Find $f(t)$ if

$$F(s) = \frac{10(s^2+119)}{(s+5)(s^2+10s+169)}.$$

Answer: $f(t) = (10e^{-5t} - 8.33e^{-5t}\sin 12t)u(t)$.

NOTE: Also try Chapter Problems 12.40(c) and (d).

Partial Fraction Expansion: Repeated Real Roots of $D(s)$

To find the coefficients associated with the terms generated by a multiple root of multiplicity r, we multiply both sides of the identity by the multiple root raised to its rth power. We find the K appearing over the factor raised to the rth power by evaluating both sides of the identity at the multiple root. To find the remaining $(r - 1)$ coefficients, we differentiate both sides of the identity $(r - 1)$ times. At the end of each differentiation, we evaluate both sides of the identity at the multiple root. The right-hand side is always the desired K, and the left-hand side is always its numerical value. For example,

$$\frac{100(s+25)}{s(s+5)^3} = \frac{K_1}{s}+\frac{K_2}{(s+5)^3}+\frac{K_3}{(s+5)^2}+\frac{K_4}{s+5}. \tag{12.65}$$

We find K_1 as previously described; that is,

$$K_1 = \left.\frac{100(s+25)}{(s+5)^3}\right|_{s=0} = \frac{100(25)}{125} = 20. \tag{12.66}$$

To find K_2, we multiply both sides by $(s + 5)^3$ and then evaluate both sides at -5:

$$\left.\frac{100(s+25)}{s}\right|_{s=-5} = \left.\frac{K_1(s+5)^3}{s}\right|_{s=-5} + K_2 + K_3(s+5)|_{s=-5}$$
$$+ \left.K_4(s+5)^2\right|_{s=-5}, \tag{12.67}$$

$$\frac{100(20)}{(-5)} = K_1 \times 0 + K_2 + K_3 \times 0 + K_4 \times 0$$
$$= K_2 = -400. \tag{12.68}$$

To find K_3 we first must multiply both sides of Eq. 12.65 by $(s + 5)^3$. Next we differentiate both sides once with respect to s and then evaluate at $s = -5$:

$$\frac{d}{ds}\left[\frac{100(s + 25)}{s}\right]_{s=-5} = \frac{d}{ds}\left[\frac{K_1(s + 5)^3}{s}\right]_{s=-5}$$

$$+ \frac{d}{ds}[K_2]_{s=-5}$$

$$+ \frac{d}{ds}[K_3(s + 5)]_{s=-5}$$

$$+ \frac{d}{ds}[K_4(s + 5)^2]_{s=-5}, \quad (12.69)$$

$$100\left[\frac{s - (s + 25)}{s^2}\right]_{s=-5} = K_3 = -100. \quad (12.70)$$

To find K_4 we first multiply both sides of Eq. 12.65 by $(s + 5)^3$. Next we differentiate both sides twice with respect to s and then evaluate both sides at $s = -5$. After simplifying the first derivative, the second derivative becomes

$$100\frac{d}{ds}\left[-\frac{25}{s^2}\right]_{s=-5} = K_1\frac{d}{ds}\left[\frac{(s + 5)^2(2s - 5)}{s^2}\right]_{s=-5}$$

$$+ 0 + \frac{d}{ds}[K_3]_{s=-5} + \frac{d}{ds}[2K_4(s + 5)]_{s=-5},$$

or

$$-40 = 2K_4. \quad (12.71)$$

Solving Eq. 12.71 for K_4 gives

$$K_4 = -20. \quad (12.72)$$

Then

$$\frac{100(s + 25)}{s(s + 5)^3} = \frac{20}{s} - \frac{400}{(s + 5)^3} - \frac{100}{(s + 5)^2} - \frac{20}{s + 5}. \quad (12.73)$$

At this point we can check our expansion by testing both sides of Eq. 12.73 at $s = -25$. Noting both sides of Eq. 12.73 equal zero when $s = -25$ gives us confidence in the correctness of the partial fraction expansion. The inverse transform of Eq. 12.73 yields

$$\mathcal{L}^{-1}\left\{\frac{100(s + 25)}{s(s + 5)^3}\right\}$$

$$= [20 - 200t^2e^{-5t} - 100te^{-5t} - 20e^{-5t}]u(t). \quad (12.74)$$

✓ASSESSMENT PROBLEM

Objective 2—Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

12.6 Find $f(t)$ if

$$F(s) = \frac{(4s^2 + 7s + 1)}{s(s + 1)^2}.$$

Answer: $f(t) = (1 + 2te^{-t} + 3e^{-t})u(t)$.

NOTE: Also try Chapter Problems 12.41(a), (b), and (d).

Partial Fraction Expansion: Repeated Complex Roots of $D(s)$

We handle repeated complex roots in the same way that we did repeated real roots; the only difference is that the algebra involves complex numbers. Recall that complex roots always appear in conjugate pairs and that the coefficients associated with a conjugate pair are also conjugates, so that only half the Ks need to be evaluated. For example,

$$F(s) = \frac{768}{(s^2 + 6s + 25)^2}. \qquad (12.75)$$

After factoring the denominator polynomial, we write

$$\begin{aligned} F(s) &= \frac{768}{(s + 3 - j4)^2(s + 3 + j4)^2} \\ &= \frac{K_1}{(s + 3 - j4)^2} + \frac{K_2}{s + 3 - j4} \\ &\quad + \frac{K_1^*}{(s + 3 + j4)^2} + \frac{K_2^*}{s + 3 + j4}. \end{aligned} \qquad (12.76)$$

Now we need to evaluate only K_1 and K_2, because K_1^* and K_2^* are conjugate values. The value of K_1 is

$$\begin{aligned} K_1 &= \left.\frac{768}{(s + 3 + j4)^2}\right|_{s=-3+j4} \\ &= \frac{768}{(j8)^2} = -12. \end{aligned} \qquad (12.77)$$

The value of K_2 is

$$\begin{aligned} K_2 &= \frac{d}{ds}\left[\frac{768}{(s + 3 + j4)^2}\right]_{s=-3+j4} \\ &= -\left.\frac{2(768)}{(s + 3 + j4)^3}\right|_{s=-3+j4} \\ &= -\frac{2(768)}{(j8)^3} \\ &= -j3 = 3\underline{/-90^\circ}. \end{aligned} \qquad (12.78)$$

From Eqs. 12.77 and 12.78,

$$K_1^* = -12, \tag{12.79}$$

$$K_2^* = j3 = 3\,\angle 90^\circ. \tag{12.80}$$

We now group the partial fraction expansion by conjugate terms to obtain

$$F(s) = \left[\frac{-12}{(s+3-j4)^2} + \frac{-12}{(s+3+j4)^2}\right]$$

$$+ \left(\frac{3\,\angle -90^\circ}{s+3-j4} + \frac{3\,\angle 90^\circ}{s+3+j4}\right). \tag{12.81}$$

We now write the inverse transform of $F(s)$:

$$f(t) = [-24te^{-3t}\cos 4t + 6e^{-3t}\cos(4t - 90^\circ)]u(t). \tag{12.82}$$

Note that if $F(s)$ has a real root a of multiplicity r in its denominator, the term in a partial fraction expansion is of the form

$$\frac{K}{(s+a)^r}.$$

The inverse transform of this term is

$$\mathcal{L}^{-1}\left\{\frac{K}{(s+a)^r}\right\} = \frac{Kt^{r-1}e^{-at}}{(r-1)!}u(t). \tag{12.83}$$

If $F(s)$ has a complex root of $\alpha + j\beta$ of multiplicity r in its denominator, the term in partial fraction expansion is the conjugate pair

$$\frac{K}{(s+\alpha-j\beta)^r} + \frac{K^*}{(s+\alpha+j\beta)^r}.$$

The inverse transform of this pair is

$$\mathcal{L}^{-1}\left\{\frac{K}{(s+\alpha-j\beta)^r} + \frac{K^*}{(s+\alpha+j\beta)^r}\right\}$$

$$= \left[\frac{2|K|t^{r-1}}{(r-1)!}e^{-\alpha t}\cos(\beta t + \theta)\right]u(t). \tag{12.84}$$

Equations 12.83 and 12.84 are the key to being able to inverse-transform any partial fraction expansion by inspection. One further note regarding these two equations: In most circuit analysis problems, r is seldom greater than 2. Therefore, the inverse transform of a rational function can be handled with four transform pairs. Table 12.3 lists these pairs.

TABLE 12.3 Four Useful Transform Pairs

Pair Number	Nature of Roots	$F(s)$	$f(t)$
1	Distinct real	$\frac{K}{s+a}$	$Ke^{-at}u(t)$
2	Repeated real	$\frac{K}{(s+a)^2}$	$Kte^{-at}u(t)$
3	Distinct complex	$\frac{K}{s+\alpha-j\beta}+\frac{K^*}{s+\alpha+j\beta}$	$2\|K\|e^{-\alpha t}\cos(\beta t+\theta)u(t)$
4	Repeated complex	$\frac{K}{(s+\alpha-j\beta)^2}+\frac{K^*}{(s+\alpha+j\beta)^2}$	$2t\|K\|e^{-\alpha t}\cos(\beta t+\theta)u(t)$

Note: In pairs 1 and 2, K is a real quantity, whereas in pairs 3 and 4, K is the complex quantity $|K|\underline{/\theta}$.

✓ASSESSMENT PROBLEM

Objective 2—Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

12.7 Find $f(t)$ if

$$F(s) = \frac{40}{(s^2 + 4s + 5)^2}.$$

Answer: $f(t) = (-20te^{-2t}\cos t + 20e^{-2t}\sin t)u(t)$.

NOTE: Also try Chapter Problem 12.41(e).

Partial Fraction Expansion: Improper Rational Functions

We conclude the discussion of partial fraction expansions by returning to an observation made at the beginning of this section, namely, that improper rational functions pose no serious problem in finding inverse transforms. An improper rational function can always be expanded into a polynomial plus a proper rational function. The polynomial is then inverse-transformed into impulse functions and derivatives of impulse functions. The proper rational function is inverse-transformed by the techniques outlined in this section. To illustrate the procedure, we use the function

$$F(s) = \frac{s^4 + 13s^3 + 66s^2 + 200s + 300}{s^2 + 9s + 20}. \quad (12.85)$$

Dividing the denominator into the numerator until the remainder is a proper rational function gives

$$F(s) = s^2 + 4s + 10 + \frac{30s + 100}{s^2 + 9s + 20}, \quad (12.86)$$

where the term $(30s + 100)/(s^2 + 9s + 20)$ is the remainder.

Next we expand the proper rational function into a sum of partial fractions:

$$\frac{30s + 100}{s^2 + 9s + 20} = \frac{30s + 100}{(s + 4)(s + 5)} = \frac{-20}{s + 4} + \frac{50}{s + 5}. \quad (12.87)$$

Substituting Eq. 12.87 into Eq. 12.86 yields

$$F(s) = s^2 + 4s + 10 - \frac{20}{s+4} + \frac{50}{s+5}. \qquad (12.88)$$

Now we can inverse-transform Eq. 12.88 by inspection. Hence

$$f(t) = \frac{d^2\delta(t)}{dt^2} + 4\frac{d\delta(t)}{dt} + 10\delta(t) - (20e^{-4t} - 50e^{-5t})u(t). \qquad (12.89)$$

✓ASSESSMENT PROBLEMS

Objective 2—Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

12.8 Find $f(t)$ if

$$F(s) = \frac{(5s^2 + 29s + 32)}{(s+2)(s+4)}.$$

Answer: $f(t) = 5\delta(t) - (3e^{-2t} - 2e^{-4t})u(t)$.

NOTE: Also try Chapter Problem 12.42(c).

12.9 Find $f(t)$ if

$$F(s) = \frac{(2s^3 + 8s^2 + 2s - 4)}{(s^2 + 5s + 4)}.$$

Answer: $f(t) = 2\frac{d\delta(t)}{dt} - 2\delta(t) + 4e^{-4t}u(t)$.

12.8 Poles and Zeros of $F(s)$

The rational function of Eq. 12.42 also may be expressed as the ratio of two factored polynomials. In other words, we may write $F(s)$ as

$$F(s) = \frac{K(s + z_1)(s + z_2)\cdots(s + z_n)}{(s + p_1)(s + p_2)\cdots(s + p_m)}, \qquad (12.90)$$

where K is the constant a_n/b_m. For example, we may also write the function

$$F(s) = \frac{8s^2 + 120s + 400}{2s^4 + 20s^3 + 70s^2 + 100s + 48}$$

as

$$F(s) = \frac{8(s^2 + 15s + 50)}{2(s^4 + 10s^3 + 35s^2 + 50s + 24)} = \frac{4(s+5)(s+10)}{(s+1)(s+2)(s+3)(s+4)}. \qquad (12.91)$$

The roots of the denominator polynomial, that is, $-p_1, -p_2, -p_3, \ldots, -p_m$, are called the **poles of $F(s)$**; they are the values of s at which $F(s)$ becomes infinitely large. In the function described by Eq. 12.91, the poles of $F(s)$ are -1, -2, -3, and -4.

The roots of the numerator polynomial, that is, $-z_1, -z_2, -z_3, \ldots, -z_n$, are called the **zeros of $F(s)$**; they are the values of s at which $F(s)$ becomes zero. In the function described by Eq. 12.91, the zeros of $F(s)$ are -5 and -10.