Middle East Technical University

Department of Electrical and Electronics Engineering

EE 531 – HW # 1

Due: October 13, 2014 (in-class)

1. (**The Monty Hall Problem**) You are a contestant in a game show in which a prize is hidden behind one of three curtains. You will win the prize if you select the correct curtain. After you pick one curtain but before that curtain is lifted, the game show host always lifts one of the remaining curtains that does not contain the prize, revealing an empty stage. Host asks if you would like to switch from your current selection to the remaining curtain. How will your chances change if you switch? (Hint: Examine each strategy separately. In each, let B be the event of winning, and A the event that the initially chosen door has the prize behind it.) Show that when you adopt a randomized strategy (you decide whether to switch or not by tossing a fair coin) the probability of winning is 1/2.

2. (Two Points on a Circle) P_1 and P_2 are two points randomly selected on the perimeter of a circle with unit circumference. Points are independent and identically chosen according to the uniform distribution. Using symmetry, argue that the arc length from P_1 to P_2 (in clockwise direction) and from P_2 to P_1 (in clockwise direction) are uniformly distributed in [0,1]. Assume that a point *X* is arbitrarily chosen on the circle before the selection of P_1 and P_2. Find the pdf of the arc length of the arc containing the point *X*. (Hint: The answer is not the uniform density.)

3. X_1 and X_2 are two independent random variables whose cdf's are denoted with $F_{X_1}(x_1)$ and $F_{X_2}(x_2)$. Define a new random variable: $U = \max(a_1 X_1, a_2 X_2 + b)$ where $a_1 \neq 0$, $a_2 \neq 0$, b are fixed and known scalars.

a) Find the cdf of U.

b) Assuming that X_1 and X_2 are continuous random variables, determine the pdf of U in terms of the derivatives of $F_{X_1}(x_1)$ and $F_{X_2}(x_2)$.

c) Find the probability that $U = a_1 X_1$ when X_1 and X_2 are both uniform random variables in the interval (0, 1).

4. Let X_1 and X_2 are independent, exponential random variables, $f_{X_k}(x_k) = \lambda_k e^{-\lambda_k x_k} u(x_k)$, $k = \{1,2\}$.

a) Find $P\{X_1 < X_2\}$ by conditioning the event of interest by $X_1 = x_1$.

b) Repeat part a) by integrating the joint pdf over the event of interest in x_1-x_2 plane.

c) Find $P\{X_1 > s+t \mid X_1 > s\}$ for $s > 0, t > 0$ and using this result find the conditional density for X_1 given $X_1 > s$.

5. Negative evidence. Suppose that the evidence of an event *B* increases the probability of a criminal's guilt; that is, if *A* is the event that the criminal is guilty, then $P(A \mid B) \geq P(A)$. Does the absence of the event *B* decrease the criminal's probability of being guilty? In other words, is $P(A \mid B^c) < P(A)$? Prove or provide a counterexample.

6. (From the textbook)

a) Exercise 1.4

b) Exercise 1.5 parts a, b, c

c) Exercise 1.6