MIDDLE EAST TECHNICAL UNIVERSITY

ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

EE 201 Circuit Theory I

Midterm Examination 2

December 6, 2013

Duration: 120 minutes

Q1	14 pts	
Q2	14 pts	
Q3	14 pts	
Q4	14 pts	
Q5	20 pts	
Q6	24 pts	
Total	100 pts	

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Section:

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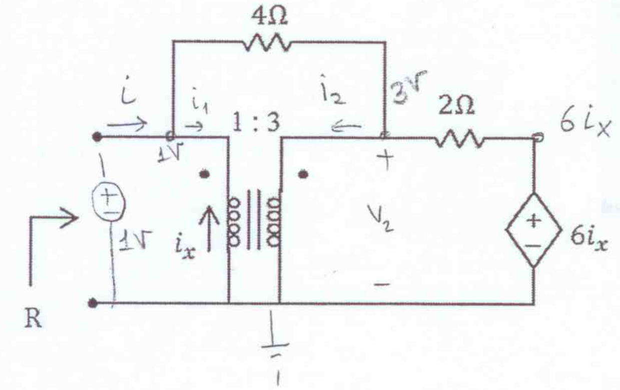
Question 1 (14 pts) Find the input resistance, R.



$$i_1 + 3i_2 = 0 \Rightarrow \boxed{i_2 = -\frac{i_1}{3}} = \frac{i_x}{3}$$

$$\frac{1V}{1} = \frac{V_2}{3} \Rightarrow \boxed{V_2 = 3V}$$

$$\underset{(3)}{\frac{3-6i_x}{2}} + \underset{(2)}{\frac{i_x}{3}} + \underset{(3)}{\frac{\cancel{2}^{1}}{\cancel{4}_{2}}} = 0$$

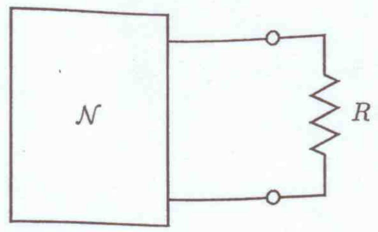
$$9 - 18i_x + 2i_x + 3 = 0$$

$$\cancel{12}_{3} = \cancel{16}_{4} i_x \Rightarrow i_x = \frac{3}{4}\ A$$

$$i = -i_x + \frac{-2}{4} = -\frac{3}{4} - \frac{2}{4} = -\frac{5}{4}\ A$$

$$\boxed{R = \frac{1}{-5/4} = -\frac{4}{5}\ \Omega} = -0.8\ \Omega$$

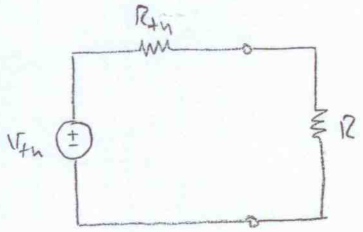
Question 2 (14 pts) The one-port circuit N in the figure below is made up of passive LTI resistors and constant independent sources. The Thevenin resistance of N is denoted by R_{TH}.
R is a passive LTI resistor. The power delivered to R is denoted by P.



(7) **a)** It is observed that for two different values of R, R_1 and R_2, P has the same value.
Show that $\sqrt{R_1R_2} = R_{TH}$.

(7) **b)** For both $R = R_o$ and $R = 4R_o$, P is 80 W. What is the maximum power that can be supplied by N?

a)



$$P_R = R\left(\frac{V_{th}}{R_{th}+R}\right)^2$$

$$P_{R_1} = P_{R_2} \Rightarrow \frac{R_1 \cancel{V_{th}^2}}{(R_{th}+R_1)^2} = \frac{R_2 \cancel{V_{th}^2}}{(R_{th}+R_2)^2}$$

$$\Rightarrow R_1(R_{th}+R_2)^2 = R_2(R_{th}+R_1)^2$$

$$\Rightarrow R_1\{R_{th}^2 + \cancel{2R_{th}R_2} + R_2^2\} = R_2\{R_{th}^2 + \cancel{2R_{th}R_1} + R_1^2\}$$

$$\Rightarrow R_{th}^2\cancel{(R_1-R_2)} = R_2R_1^2 - R_1R_2^2 = R_1R_2\cancel{(R_1-R_2)}$$

$$\Rightarrow R_{th}^2 = R_1R_2 \quad \Rightarrow \quad \boxed{R_{th} = \sqrt{R_1R_2}}$$

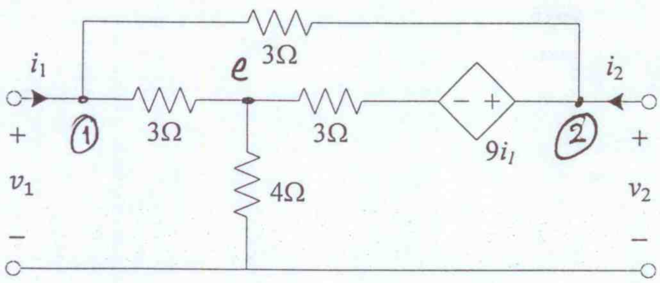
b) $$R_{th} = \sqrt{R_o(4R_o)} = 2R_o$$

$$P_{max} = P_{R_{th}} = R_{th}\left(\frac{V_{th}}{2R_{th}}\right)^2 = \frac{V_{th}^2}{4R_{th}} = \frac{V_{th}^2}{8R_o}$$

$$80 = P_{R_o} = R_o\frac{V_{th}^2}{(2R_o+R_o)^2} = \frac{V_{th}^2}{9R_o}$$

$$\Rightarrow V_{th}^2 = 720R_o \quad \Rightarrow \quad P_{max} = \frac{720R_o}{8R_o} = \boxed{90\text{ W}}$$

Question 3 (14 pts) Find the resistance (open circuit) parameters of the two-port circuit shown in the figure.



1^{st} method

$$\frac{e-v_1}{3}+\frac{e+9i_1-v_2}{3}+\frac{e}{4}=0 \Rightarrow e=\frac{4}{11}v_1+\frac{4}{11}v_2-\frac{36}{11}i_1$$

(1) $$i_1=\frac{v_1-e}{3}+\frac{v_1-v_2}{3}$$

(2) $$i_2=\frac{v_2-v_1}{3}+\frac{v_2-e-9i_1}{3}$$

(1') $$i_1=\frac{6}{11}v_1+\frac{12}{11}i_1-\frac{5}{11}v_2$$

(2') $$i_2=-\frac{5}{11}v_1+\frac{6}{11}v_2-\frac{5}{11}i_1$$

$$\Rightarrow \quad v_1=9i_1+5i_2$$
$$v_2=6i_2+11i_1$$

$$\therefore \quad R=\begin{bmatrix}9 & 5\\ 11 & 6\end{bmatrix}$$

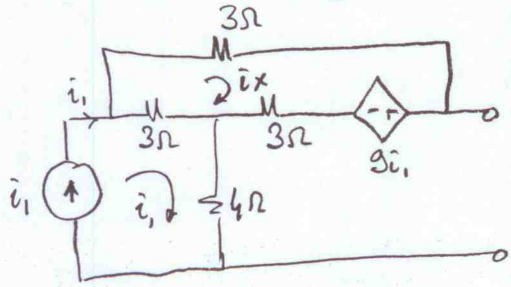
$r_{11}=9\Omega$ $\quad r_{21}=11\Omega$

$r_{12}=5\Omega$ $\quad r_{22}=6\Omega$

$$\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}9 & 5\\ 11 & 6\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix}$$

2nd method:

$i_2=0$ and apply i_1:



$$9i_x+9i_1-3i_1=0$$

$$\Rightarrow i_x=-\frac{2}{3}i_1$$

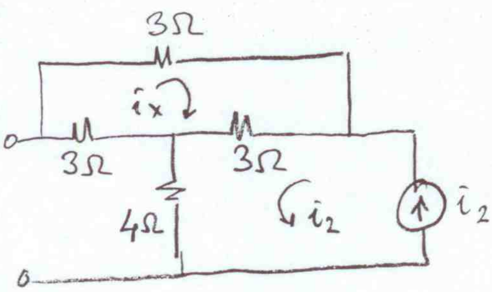
$$v_1=7i_1-3i_x=9i_1$$

$$\Rightarrow r_{11}=\left.\frac{v_1}{i_1}\right|_{i_2=0}=9\Omega$$

$$v_2=9i_1+3i_x+4i_1=11i_1$$

$$\Rightarrow r_{21}=\left.\frac{v_2}{i_1}\right|_{i_2=0}=11\Omega$$

$i_1=0$ and apply i_2:



$$9i_x+3i_2=0$$

$$i_x=-\frac{1}{3}i_2$$

$$v_1=-3i_x+4i_2=5i_2$$

$$\Rightarrow r_{12}=\left.\frac{v_1}{i_2}\right|_{i_1=0}=5\Omega$$

$$v_2=7i_2+3i_x=6i_2$$

$$\Rightarrow r_{22}=\left.\frac{v_2}{i_2}\right|_{i_1=0}=6\Omega$$

Question 4 (14 pts) A linear time-invariant (LTI) resistive 3-terminal two-port circuit with conductance (short circuit) parameters "g_{11} = 5 mho, g_{12} = –2 mho, g_{21} = 2 mho, g_{22} = 4 mho" is to be designed. Available elements are LTI resistors, LTI dependent sources and independent sources.

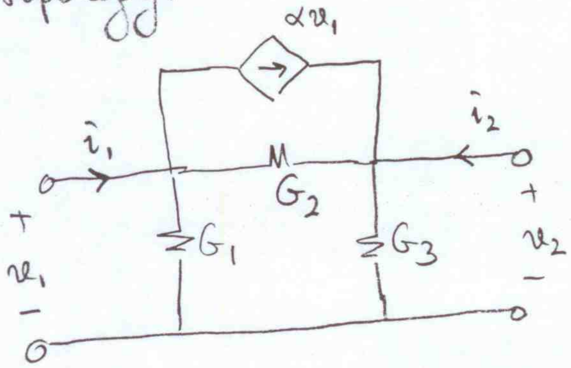
The cost for each type of element is given below:

An LTI resistor: 2 CTMU (Circuit Theory Money Unit),
An LTI dependent source: 4 CTMU,
An independent source: 6 CTMU.

Obtain a two-port circuit with minimum cost.

Since $g_{12} \neq g_{21}$ circuit is not reciprocal and must contain a dependent source.

A solution can be obtained by using the following topology:



$$i_1 = \alpha v_1 + G_1 v_1 + G_2 (v_1 - v_2)$$

$$i_2 = -\alpha v_1 + G_2 (v_2 - v_1) + G_3 v_2$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} G_1 + G_2 + \alpha & -G_2 \\ -G_2 - \alpha & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

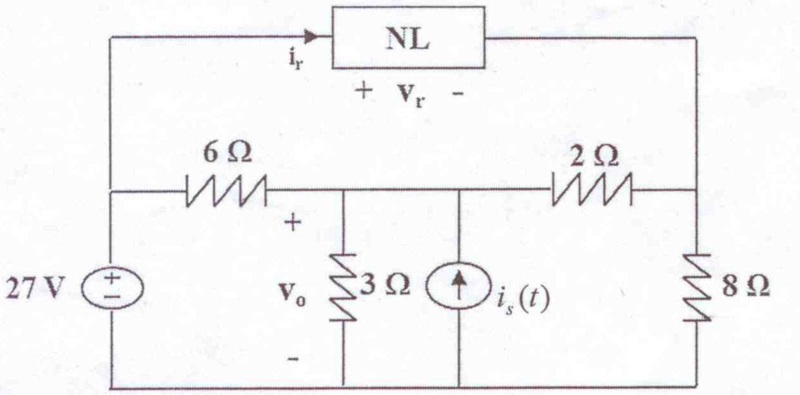
Hence

$G_2 = 2\,℧$; $G_3 = 2\,℧$

$\alpha = -4\,℧$; $G_1 = 7\,℧$

will satisfy the required short cct parameters.

Question 5 (20 pts) Consider the circuit given below.

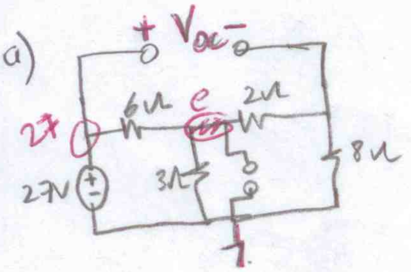


$$i_r = \begin{cases} \frac{3}{4}v_r^2, & v_r \ge 0 \\ 0, & v_r < 0 \end{cases}$$

a) For $i_s(t) = 0$, find $\mathbf{v_o}$.

b) For $i_s(t) = 0.1\cos(1000\pi t)\ A$, find the approximate $\mathbf{v_o}(t)$.

a)



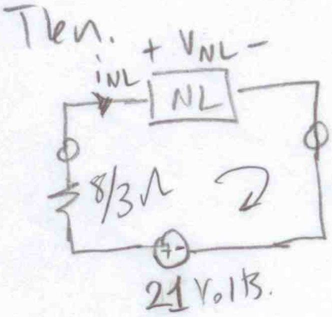
i) $R_{Th} = (\overbrace{6\|3}^{2}+2)\|8 = 4\|8 = 4\times(1\|2) = 4\cdot\frac{2}{3} = \frac{8}{3}\,\Omega$

ii) $V_{oc} = ?$

×30/ $\frac{e-27}{6}+\frac{e}{3}+\frac{e}{10}=0 \rightarrow e = \frac{27.5}{5+10+3} = \frac{15}{2}$ Volts.

$V_{oc} = 27 - e\frac{8}{8+2} = 27 - \frac{15}{2}\cdot\frac{4}{5} = 21$ Volts.

Then.



→ KVL/ $-21 + \frac{8}{3}\overbrace{i_{NL}}^{\frac{3}{4}\vartheta_{NL}^2} + V_{NL} = 0 \rightarrow 2\vartheta_{NL}^2 + \vartheta_{NL} - 21 = 0$

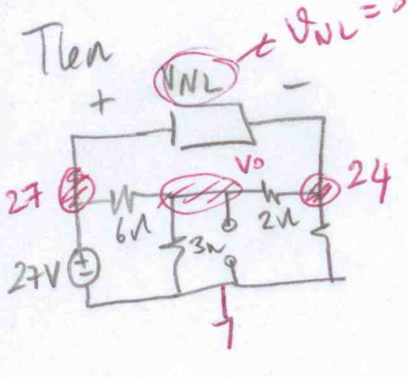
→ provided that $\vartheta_{NL} > 0$

$\vartheta_{NL} = \frac{-1 \mp \sqrt{1+168}}{4}$

$\vartheta_{NL} = \{3, -3.5\}$

Operating point.

Then



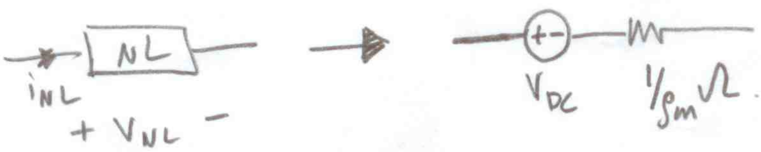
KCL at Vo: $\frac{V_o}{3} + \frac{V_o-27}{6} + \frac{V_o-24}{2} = 0 \rightarrow V_o = \frac{\cancel{27}^{9}}{\cancel{6}_{2}} + \frac{24}{2}$

$\boxed{V_o = \frac{33}{2}\text{ Volts}}$

a) 10 pts

b) 10 pts.

b) Using small signal approximation, that is



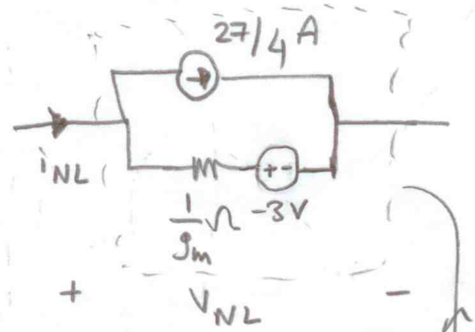
$$g_m = \left.\frac{di_{NL}}{dv_{NL}}\right|_{V_{NL}=3V} = \frac{3}{4}\cdot 2v_{NL} = \frac{9}{2}\,\mho \quad ;$$

Hence

$$i_{NL} = \frac{3}{4}v_{NL}^2 = \frac{27}{4} + g_m(v_{NL}-3) + \frac{1}{2!}\frac{3}{2}(v_{NL}-3)^2$$ } Taylor Series expansion around $v_{NL}=3$

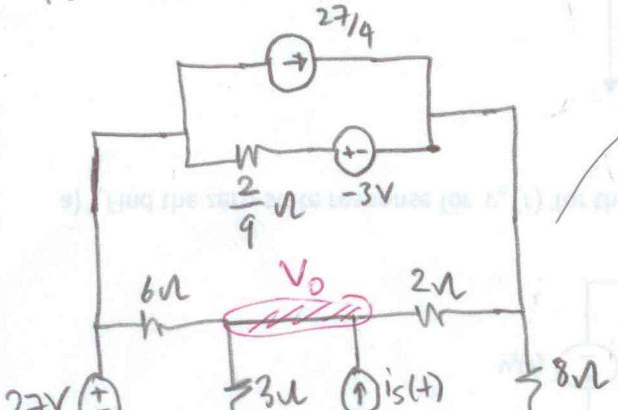
Neglecting 2nd order term (small signal approximation)

$$i_{NL} = \frac{27}{4} + g_m(v_{NL}-3) \longrightarrow$$

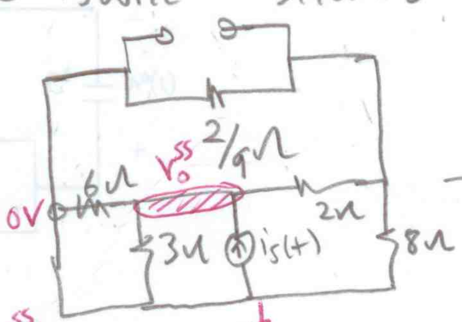


Equivalent small signal model.

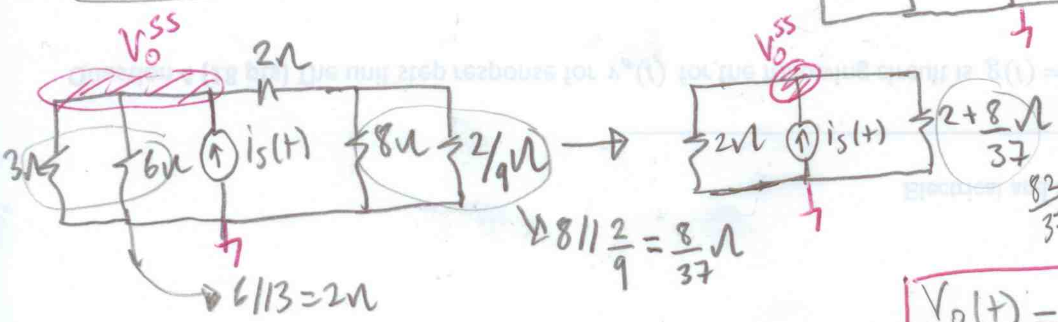
Then we have.



We have already solved for the DC part solution in part (a). Then superposition circuit for any AC source becomes



$\longrightarrow$ $0.1\cos(1000\pi t)$



$$V_0^{SS} = i_s(t)\cdot\overbrace{\left(2 // \frac{82}{37}\right)}^{41/39\,\Omega}$$

$$\boxed{V_0^{SS}(t) = \frac{4.1}{39}\cos(1000\pi t)}$$

$$\boxed{V_0(t) = 33/2 + 4.1/39\,\cos(1000\pi t)\,V}$$

Question 6 (24 pts) Obtain and sketch the transfer $(v_2 - v_1)$ and the input $(i - v_1)$ characteristics.