Name:

Department of Electrical and Electronics Engineering
Middle East Technical University

Question 1 (20 pts) For some lumped circuit N, made of single branch elements, a fundamental loop matrix B is given below.

$$\begin{array}{ccccc|cccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \longrightarrow \text{branch numbers} \\ B= & 0 & 0 & -1 & 1 & 1 & 0 & 0 & \\ & -1 & -1 & 0 & 1 & 0 & 1 & 0 & \\ & 0 & -1 & 0 & 1 & 0 & 0 & 1 & \\ & & \underline{\underline{F}} & & & & \underline{\underline{I}} & & \end{array}$$

a) Obtain a possible graph of circuit N.

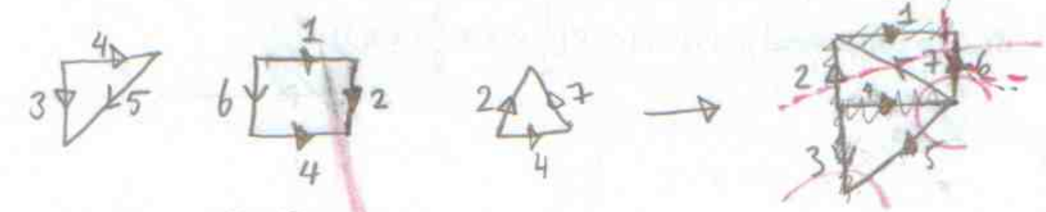
b) Write the corresponding fundamental cutset matrix Q.

c) Let $\hat{N}$ be the dual circuit of N. Some of the branch currents (in Amps) and voltages (in Volts) of dual circuit $\hat{N}$ are given below. Find the missing currents and voltages.

$\hat{i}_1 = ?$	$\hat{i}_2 = 3$	$\hat{i}_3 = ?$	$\hat{i}_4 = 1$	$\hat{i}_5 = 1$	$\hat{i}_6 = 6$	$\hat{i}_7 = ?$
$\hat{v}_1 = ?$	$\hat{v}_2 = -5$	$\hat{v}_3 = -1$	$\hat{v}_4 = ?$	$\hat{v}_5 = ?$	$\hat{v}_6 = ?$	$\hat{v}_7 = 3$

10pt

a)



b) $$Q = \begin{array}{cccc|ccc} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} & \textcircled{5} & \textcircled{6} & \textcircled{7} \\ 1 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 1 & 0 & 0 & 0 & +1 & +1 \\ 0 & 0 & 1 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & -1 \\ & & & & \multicolumn{3}{c}{\underbrace{\hphantom{-1\;-1\;-1}}_{-\underline{\underline{F}}^T}} \end{array}$$

10pt.

c) Then N satisfies

$v_1 = ?$ ___	$v_2 = 3$	$v_3 = ?$ ___	$v_4 = 1$	$v_5 = 1$	$v_6 = 6$	$v_7 =$ ___
$i_1 =$ ___	$i_2 = -5$	$i_3 = -1$	$i_4 =$ ___	$i_5 =$ ___	$i_6 =$ ___	$i_7 = 3$

From graph $\longrightarrow$

$$v_1 = -v_2 + v_4 + v_6 = 4V \longrightarrow \hat{i}_1 = 4A$$
$$v_3 = v_4 + v_5 = 2V \longrightarrow \hat{i}_3 = 2A$$
$$v_7 = +v_2 - v_4 = +2V \longrightarrow \hat{i}_7 = +2A$$

From graph $\longrightarrow$

$$i_1 = i_2 + i_7 = -2A. \longrightarrow \hat{v}_1 = -2\,v$$
$$i_4 = -i_2 - i_3 = 6A. \longrightarrow \hat{v}_4 = 6V$$
$$i_5 = -i_3 = 1A. \longrightarrow \hat{v}_5 = 1V$$
$$i_6 = -i_1 = +2A. \longrightarrow \hat{v}_6 = 2V$$

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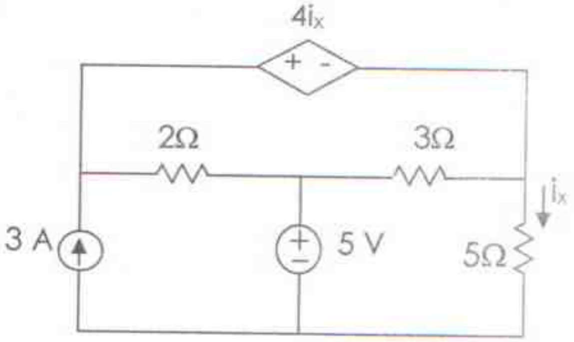
Question 2 (20 pts) Given the following circuit,

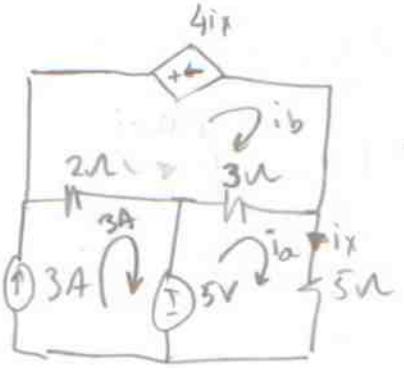


a) Obtain the mesh equation(s) and put into the matrix form.

b) Obtain the node equation(s).

c) Find the power supplied / absorbed by the dependent source.

a)



$$\longrightarrow \text{KVL } i_b \rightarrow +4\overset{i_a}{i_x} + 2(i_b - 3) + 3(i_b - i_a) = 0$$

$$\text{KVL } i_a \rightarrow 5i_a - 5 + 3(i_a - i_b) = 0$$

$$\begin{bmatrix} 1 & 5 \\ 8 & -3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

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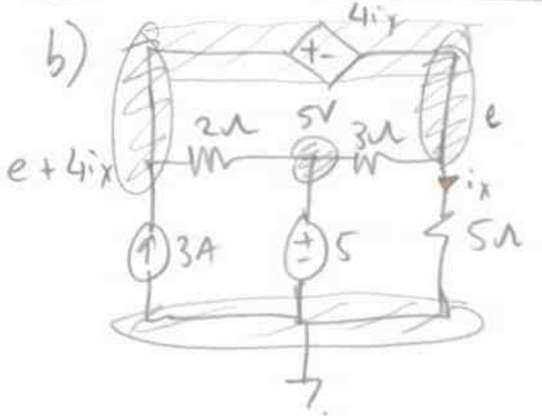
b)



KCL supernode →

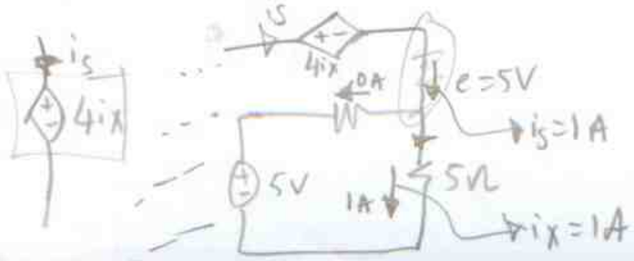
$$\frac{e}{5} + \frac{e-5}{3} + \frac{e + 4\overset{e/5.}{i_x} - 5}{2} - 3 = 0$$

$$\left(\frac{1}{5} + \frac{1}{3} + \frac{9}{10}\right)e = \frac{5}{3} + \frac{5}{2} + 3$$

$$e = \frac{50 + 75 + 90}{6 + 10 + 27} = \frac{215}{43}$$

$$e = 5V$$

c) $P_{4i_x\,source} = i_s \cdot 4i_x \longrightarrow P_{4i_x\,Source} = 1 \cdot (4 \cdot 1) = 4\,W$ absorbed.



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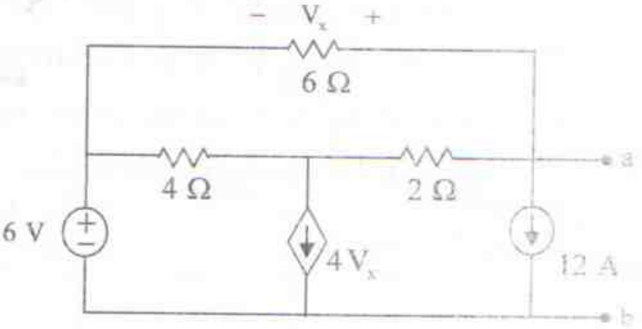
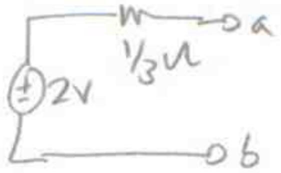
Question 3 (20 pts)

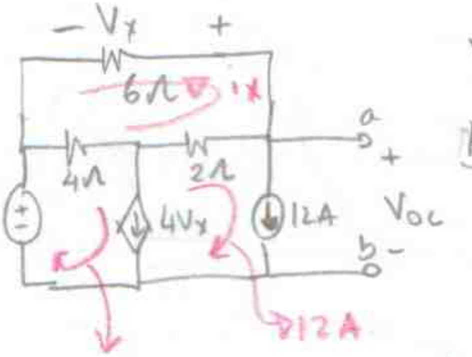
Find the Thevenin equivalent circuit to the left of terminals a and b.



Answer:



a) V_{oc} :



$4V_x + 12 = 12(2i_x + 1)$

$= 24i_x + 12$

$V_x = 6i_x$

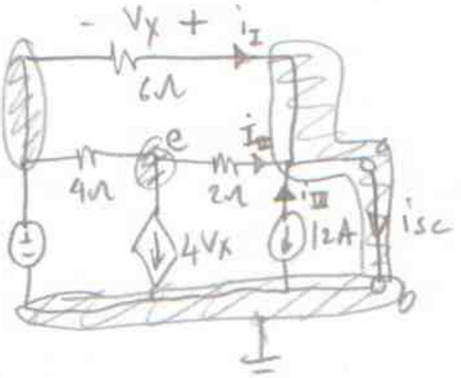
KVL i_x loop:

$6i_x + 4(i_x + 24i_x + 12) + 2(i_y + 12) = 0$

$i_x = \frac{-48-24}{100+6+2} = \frac{-72}{108} = \frac{-2}{3}$

$\boxed{V_x = -4V} \rightarrow \boxed{V_{oc} = 6 + V_x = 2V}$

i_{sc} :



KCL @ e:

$\frac{e-6}{4} + \frac{e}{2} + 4V_x = 0 \rightarrow e = \frac{24 + 3/2}{3/4}$

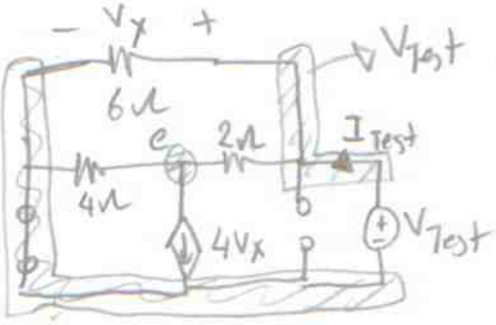
$\overset{\nwarrow}{-6.}$

$\boxed{e = 34V}$

$\boxed{i_{sc} = i_I + i_{II} + i_{III} = 1 + 17 + (-12) = 6A.}$

$R_{Th} = \frac{V_{oc}}{i_{sc}} = \frac{2}{6} = 1/3$

Test Voltage Method for R_{Th}:



$V_x = V_{Test}.$

$\frac{e}{4} + \frac{e - V_{Test}}{2} + 4\overset{V_{Test}}{V_x} = 0 \rightarrow e = -\frac{3.5}{0.75}V_{Test}$

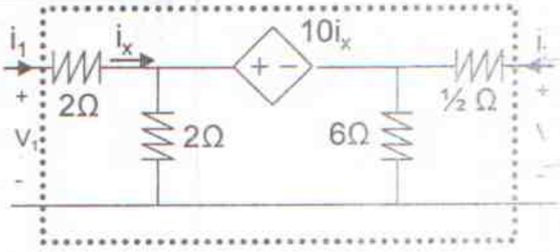
$e = -\frac{14}{3}V_{Test}$

$I_{test} = \frac{V_x}{6} + \frac{V_{Test} - e}{2} = V_{Test}\underbrace{\left(\frac{1}{6} + \frac{1}{2} + \frac{14}{6}\right)}_{3}$

$\frac{V_{Test}}{I_{Test}} = 1/3 \rightarrow \boxed{R_{Th} = 1/3 \Omega}$

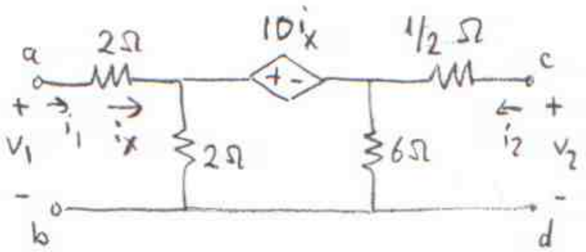
Question 4 (20 pts)

a) Find the *resistance (open circuit) parameters* of the following two port.



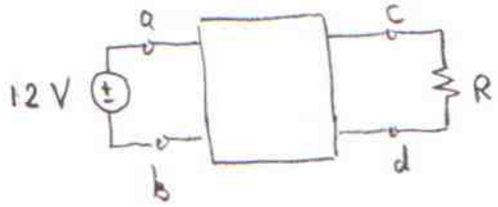
b) The two port shown in part (a) is used in the following configuration. Determine R_L so that it absorbs maximum power. Also compute this power.

a)



Find the resistance (open circuit) parameters.

b)

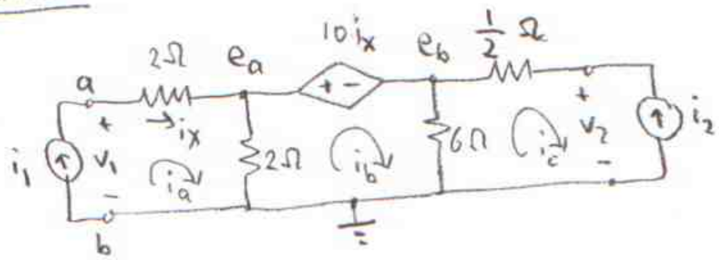


Determine R so that it absorbs the maximum power.

Also compute this power.

Solution

a)



$i_x = i_1$

$e_a = e_b + 10i_x = e_b + 10i_1$

$$-i_1 + \frac{1}{2}\underset{\underset{e_b+10i_1}{\uparrow}}{e_a} + \frac{1}{6}e_b - i_2 = 0 \Rightarrow \frac{2}{3}e_b = -4i_1 + i_2 \Rightarrow e_b = -6i_1 + \frac{3}{2}i_2$$

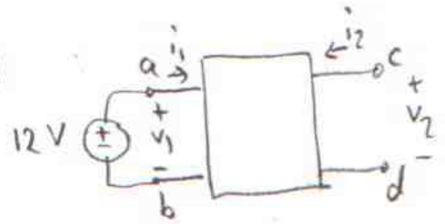
$$e_a = 4i_1 + \frac{3}{2}i_2$$

$$v_1 = 2i_1 + e_a = 6i_1 + \frac{3}{2}i_2$$

$$v_2 = e_b + \frac{1}{2}i_2 = -6i_1 + \frac{3}{2}i_2 + \frac{1}{2}i_2 = -6i_1 + 2i_2$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 6 & 3/2 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

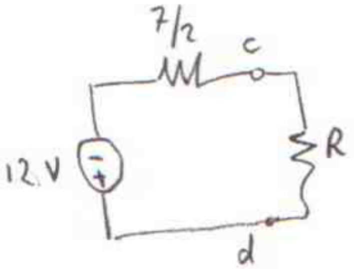
b)



$v_1 = 12$ V

$$6i_1 + \frac{3}{2}i_2 = 12 \Rightarrow i_1 = 2 - \frac{1}{4}i_2$$

$$v_2 = -6\left(2 - \frac{1}{4}i_2\right) + 2i_2 = -12 + \frac{3}{2}i_2 + 2i_2 = -12 + \frac{7}{2}i_2$$



$$R = \frac{7}{2}\,\Omega$$

$$P = \frac{144}{4 \cdot \frac{7}{2}} = \frac{72}{7} \text{ W}$$

$i_a = i_1$, $i_c = -i_2$, $i_x = i_a = i_1$

$$\underset{\overset{\uparrow}{i_1}}{10i_x} + 6(i_b - \underset{\overset{\uparrow}{-i_2}}{i_c}) + 2(i_b - \underset{\overset{\uparrow}{i_1}}{i_a}) = 0 \Rightarrow 10i_1 + 8i_b + 6i_2 - 2i_1 = 0$$

$$8i_b = -8i_1 - 6i_2$$

$$i_b = -i_1 - \frac{3}{4}i_2$$

$$v_1 = 2i_a + 2(i_a - i_b) = 4i_1 + 2i_1 + \frac{3}{2}i_2 = 6i_1 + \frac{3}{2}i_2$$

$$v_2 = 6(i_b - i_c) - \frac{1}{2}i_c = -6i_1 - \frac{9}{2}i_2 + \frac{13}{2}i_2 = -6i_1 + 2i_2$$

Question 5 (10 pts) Using the voltage and current values indicated on the circuit of Figure 1, find the power supplied/absorbed by the independent current source of the circuit of Figure 2.

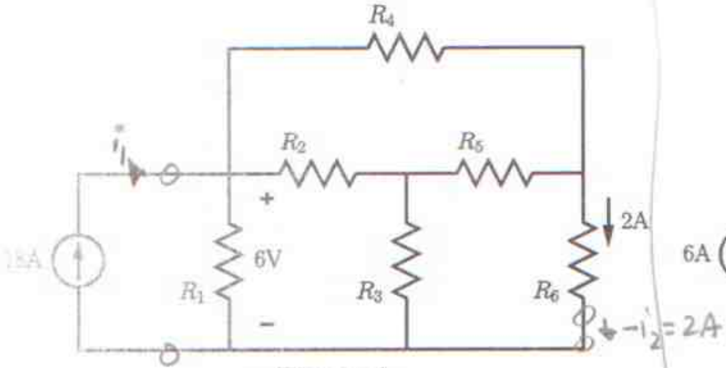


Figure 1

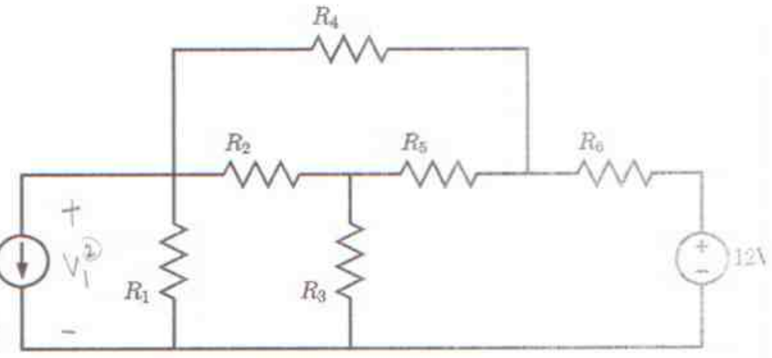
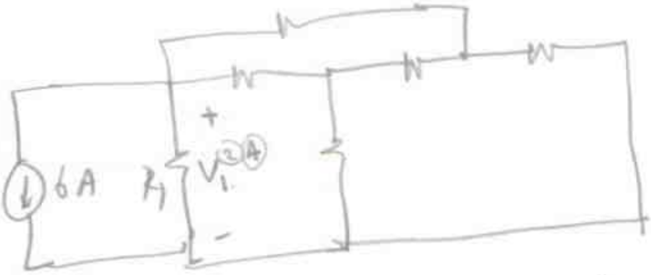


Figure 2

Find $V_1^{②}$ then power of 6A source

Ⓐ: Kill 12V source first →

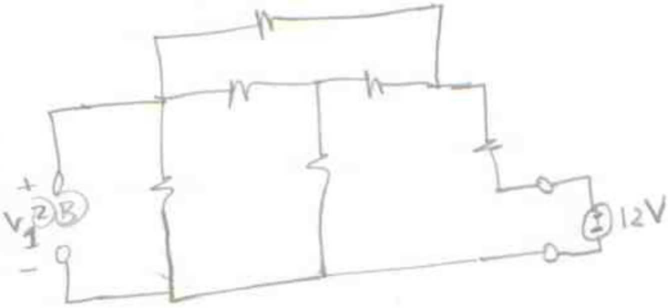
Ⓐ:



Then $V_1^{②Ⓐ} = 6 \cdot \frac{-6}{18} = -2V$ (by linearity)

Ⓑ Kill 6A source Then →

Ⓑ:



$V_1^{②Ⓑ} = \underset{\uparrow\, 2/18}{h_{12}}\, 12$

$= \frac{4}{3} V$

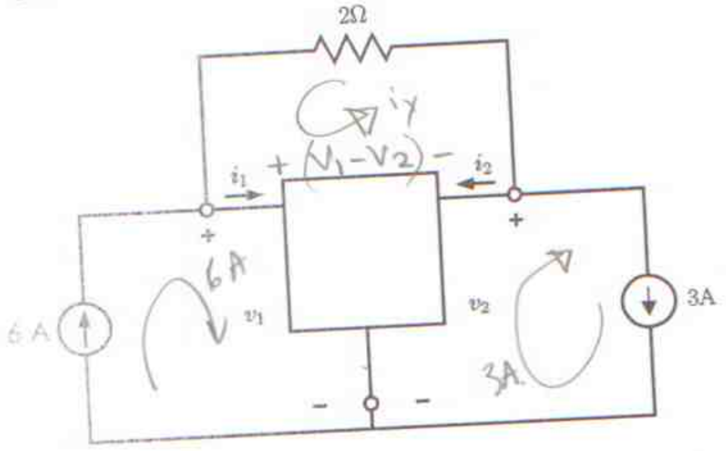
(due to reciprocity $h_{12} = -h_{21} = \frac{-2}{18}$)

$V_1^{②} = V_1^{②Ⓐ} + V_1^{②Ⓑ} = -2 + \frac{4}{3} = -2/3$

$P_{absorbed} = V_1^{②} \cdot 6 = -4W \longrightarrow \boxed{4W \text{ supplied}}$

Name:

Question 6 (10 pts) Find the powers delivered to the resistor and to the three-terminal element.



$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

#meshes − #current sources

$\Rightarrow 3-2=1$ unknown!

KVL in i_x loop:

$-4i_1 \leftarrow (i_x+6)$

$$2i_x + V_1 - V_2 = 0 \longrightarrow 2i_x + 4\underbrace{(i_1+i_2)}_{3} = 0$$

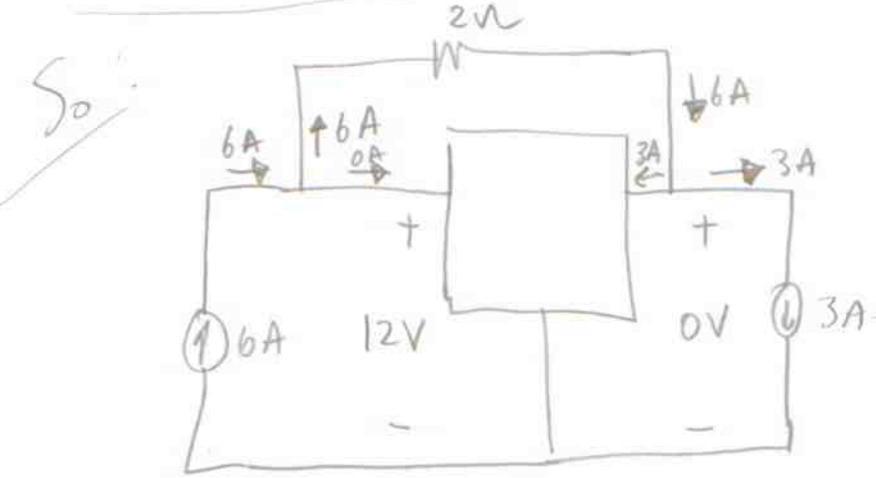
$4i_2 \leftarrow (-i_x-3)$

$$\boxed{i_x = -6}$$

$i_1 = i_x + 6 = 0$ A , $V_1 = 12$V

$i_2 = -i_x - 3 = 3$ A , $V_2 = 0$V

So



$P_{2Ω} = 6^2 . 2 = 72$ W

$P_{comp} = i_1 v_1 + i_2 v_2$

$= 0.12 + 3.0$

$= 0$ W.