



Course Information

Course Code	2360429				
Course Section	1				
Course Title	INTRODUCTION TO ERGODIC THEORY				
Course Credit	3				
Course ECTS	6.0				
Course Catalog Description	Overview Of measure spaces. Measure-preserving transformations. Recurrence and the Poincaré recurrence theorem. Ergodic transformations. The Birkhoff ergodic theorem and von Neumann mean ergodi? theorem. Applications of ergodic theorems, Weak and strong mixing transformations. Isomorphism and factors of measure-preserving transformations. Basics of topological Dynamics. Existence of invariant measures for continuous transformations and the Kryloff-Bogoliouboff theorem. Various selected advanced topics such as ag measure-theoretic entropy etc.				
Prerequisites	Students must complete one of the following sets to take this course.				
	<table border="1"> <thead> <tr> <th>Set</th> <th>Prerequisites</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2360419</td> </tr> </tbody> </table>	Set	Prerequisites	1	2360419
Set	Prerequisites				
1	2360419				
Schedule	Monday , 13:40 - 14:30, M102 Friday , 15:40 - 17:30, M102				

Instructor Information

Name/Title	Assist.Prof.Dr BURAK KAYA
Office Address	M126
Email	burakk@metu.edu.tr
Personal Website	http://blog.metu.edu.tr/burakk
Social Media	https://twitter.com/burakkayaburak
Office Phone	
Office Hours	TBA

Course Objectives

At the end of this course, the student will learn:

- basic theory and examples of measure-preserving dynamical systems and ergodic systems,
- recurrence theorems, ergodic theorems and their applications, and
- some selected advanced topics regarding measure-preserving dynamical systems.

Course Learning Outcomes

Student, who passed the course satisfactorily will be able to:

- recognize and build various measure-preserving transformations,
- recognize the notion of ergodicity and understand the behavior of ergodic dynamical systems,
- use recurrence and ergodic theorems to prove results,
- understand the relationship between measure-preserving dynamical systems such as isomorphism, conjugacy and factor relations,
- recognize and build various types of topological dynamical systems,
- understand the behavior of various types of topological dynamical systems, and
- calculate the entropy of a measure-preserving transformation.

Program Outcomes Matrix

Undergraduate



Program Outcomes	Level of Contribution			
	0	1	2	3
1 Acquires mathematical thinking skills (problem solving, generating ways of thinking, forming correspondence, generalizing etc.) and can use them in related fields.				X
2 Can produce innovative thoughts and products.				X
3 Can design mathematics related problems, devise solution methods and apply them when appropriate.			X	
4 Has a comprehension of mathematical symbols, concepts together with the interactions among them and can express his/her solutions similarly.				X
5 Has a command of Turkish and English languages so that he/she can actively communicate (read, write, listen and speak).		X		
6 Contributes to solving global, environmental and social problems either individually or as being part of a social group.	X			
7 Respects ethical values and rules; applies them in professional and social issues.		X		
8 Can work cooperatively in a team and also individually.		X		
9 Is responsive to life-long learning, improving his/her skills and abilities			X	
10 Comprehends necessity of knowledge, can define it and acquires it; uses knowledge effectively and shares it with others				X

0: No Contribution 1: Little Contribution 2: Partial Contribution 3: Full Contribution

Instructional Methods

Lectures will be held in person, in a classical classroom environment.

Tentative Weekly Outline

Week	Topic	Relevant Reading	Assignments
1	An overview of measure spaces, function spaces and some examples that are to be used. The (statement of the) existence of Haar measure and some examples		
2	Measure-preserving transformations and examples (such as circle rotations, circle doubling map, odometers, Bernoulli shifts, Gauss map etc.)		
3	Associated isometry of a measure-preserving transformation. Recurrence and the Poincaré recurrence theorem		
4	Ergodicity and examples of ergodic transformations (such as irrational rotations, circle doubling, Bernoulli shifts etc.)		
5	The maximal ergodic theorem and the Birkhoff ergodic theorem		
6	The Birkhoff ergodic theorem and Von Neumann's mean ergodic theorem		



Week	Topic	Relevant Reading	Assignments
7	Some applications of ergodic theorems: Borel's theorem on normal numbers, Weyl equidistribution theorem, the existence of Khinchin's constant		
8	Weak mixing and strong mixing transformations, examples and non-examples		
9	Isomorphism, conjugacy and factors of measure-preserving transformations and examples		
10	Isomorphism, conjugacy and factors of measure-preserving transformations and examples		
11	Basics of topological dynamics: Minimality, topological transitivity, the non-wandering set and examples.		
12	Invariant measures for continuous transformations and the Kryloff-Bogoliouboff theorem. The (statement of the) ergodic decomposition theorem.		
13	Selected advanced topics: Entropy of a measure-preserving transformation		
14	Selected advanced topics: Calculation of entropy of various measure-preserving transformations. The statement of Ornstein's theorem		

Course Textbook(s)

We shall be using my lecture notes that will be uploaded to ODTÜ Class throughout semester. That said, the lecture notes will be mainly based on the textbooks

- [Invitation to Ergodic Theory](#)
by C. E. Silva, Softcover ISBN: 978-0-8218-4420-5
- [An Introduction to Ergodic Theory](#)
by Peter Walters, Softcover ISBN: 978-0-387-95152-2

Supplementary Readings / Resources / E-Resources

Readings

In addition to the lecture notes and the textbook on which the lectures notes are based, the following textbook may sometimes be used

- [Ergodic Theory with a view towards Number Theory](#)
by Manfred Einsiedler and Thomas Ward, Softcover ISBN: 978-1-4471-2591-4

Assessment of Student Learning

Assessment	Dates or deadlines
There will be three midterm exams each out of 80 points. Each midterm will have %20 weight in your overall score that is to be used for letter grades.	TBA
I will use Gradescope to grade your exams, unless you prefer otherwise in which case you should let me know in advance.	



Assessment

Dates or deadlines

There will be one **final exam** out of 100 points. The final exam will have **%40 weight** in your overall score that is to be used for letter grades.

I will use **Gradescope** to grade your exams, unless you prefer otherwise in which case you should let me know in advance.

TBA

Course Grading

Deliverable	Grade Points
Midterm I	20
Midterm II	20
Midterm III	20
Final Exam	40
Total	100

Course Policies

Class Attendance

After the add-drop week, starting from the beginning of 3rd week until the end of 12th week of classes, **attendance will be taken every lecture hour of every week.**

You need to **attend 70% of the classes, more precisely, you need to attend 70% of the classes starting from the beginning of 3rd week until the end of 12th week.** In accordance with [Middle East Technical University Rules and Regulations Governing Undergraduate Studies](#) Article 24(5)(b)(1), you shall **receive the NA grade because of not fulfilling the attendance requirement.**

The rationale behind this strict attendance requirement is the following. This course is an advanced course, with another advanced course as its prerequisite, that covers topics and mathematical objects which are not even tangentially mentioned in core courses. Consequently, the material that you shall learn in this class is going to be entirely new, even to those who have strong background and have taken various elective courses. Since attending classes is vital to learning and digesting the course material, such a strict attendance policy has been deemed appropriate.

Class Participation

While class participation is **not formally graded**, it is **highly encouraged** since "mathematics is not a spectator sport" as Pólya once said.

Make up for Exams and Assignments

No make-ups will be given without an official report. This policy is non-negotiable. If you are going to miss an exam because of extraordinary conditions, which may not be documented via reports, you should contact me *in advance*.

Final Exam Entrance Conditions

In order to be able to take the final exam, you need to **attend 70% of the classes starting from the beginning of 3rd week until the end of 12th week.** In accordance with [Middle East Technical University Rules and Regulations Governing Undergraduate Studies](#) Article 24(5)(b)(1) and 24(5)(b)(2), you shall **receive the NA grade in the case that you do not fulfill the attendance requirement.**

Information for Students with Disabilities

Students who experience difficulties due to their disabilities and wish to obtain academic adjustments and/or auxiliary aids must contact ODTU Disability Support Office and/or course instructor and the advisor of students with disabilities at academic departments (for the list: <http://engelsiz.metu.edu.tr/en/advisor-students-disabilities>) as soon as possible. For detailed information, please visit the website of Disability Support Office: <https://engelsiz.metu.edu.tr/en/>



Academic Honesty

The METU Honour Code is as follows: *"Every member of METU community adopts the following honour code as one of the core principles of academic life and strives to develop an academic environment where continuous adherence to this code is promoted. The members of the METU community are reliable, responsible and honourable people who embrace only the success and recognition they deserve, and act with integrity in their use, evaluation and presentation of facts, data and documents."*