PLEASE WRITE YOUR NAME CLEARLY USING CAPITAL LETTERS		
FULL NAME	STUDENT ID	DURATION 140 MINUTES
5 QUESTIONS		TOTAL 100(+20 BONUS) POINTS

By signing below, I pledge that I will write this examination as my own work and without the assistance of others or the usage of unauthorized material or information. I understand that possession of any kind of electronic device during the exam is prohibited. I also understand that not obeying the rules of the examination will result in immediate cancellation and disciplinary procedures.

Signature ....

(20 pts) 1. Let κ be an uncountable singular cardinal. Show that there exists a κ-Aronszajn tree, i.e. a tree $(T,<)$ such that $\text{height}(T)=\kappa$, $|\text{level}_\alpha(T)|<\kappa$ for every $\alpha<\kappa$ and every branch of T has size $<\kappa$.

(20 pts) 2. Show that $\diamondsuit$ implies CH.

(20+20 pts) 3. Let $I\subseteq\mathbb{R}$ be a bounded non-empty open interval. Consider the poset $(\mathbb{P}_I,\subseteq)$ where

$$\mathbb{P}_I=\{U\subseteq\mathbb{R}:\ U \text{ is non-empty open and } \overline{U}\subseteq I\}$$

a) Show that the poset $(\mathbb{P}_I,\subseteq)$ is ccc.

For the next part of this question, you are given that $\bigcap_{U\in G}\overline{U}$ is non-empty for any filter $G\subseteq\mathbb{P}_I$ and any bounded non-empty open interval $I\subseteq\mathbb{R}$.

b) Assuming Martin's axiom, prove that for every $\aleph_0\le\kappa<2^{\aleph_0}$ and every sequence $\{U_\alpha\}_{\alpha<\kappa}$ of open dense subsets of $\mathbb{R}$, we have that $\bigcap_{\alpha<\kappa}U_\alpha$ is dense in $\mathbb{R}$.

(20 pts) 4. Let $M\models\text{ZFC}$ be a countable transitive model and consider the forcing poset $(\mathbb{P},\le,\mathbf{1})=\left(\text{Fin}\left(\omega_1^M,\mathcal{P}(\omega)^M\right),\supseteq,\emptyset\right)\in M$. Let G be a $\mathbb{P}$-generic filter over M. Show that $M[G]\models\mathcal{P}(\omega)^M$ is countable.

(20 pts) 5. Let $M\models\text{ZFC}$ be a countable transitive model, $(\mathbb{P},\le,\mathbf{1})\in M$ be a forcing poset and $E\subseteq\mathbb{P}$ be such that $E\in M$. Let $G\subseteq\mathbb{P}$ be a $\mathbb{P}$-generic filter over M. Show that

$$\text{either }\ \exists q\in G\ \forall r\in E\ q\perp r\ \text{ or }\ G\cap E\neq\emptyset$$