PLEASE WRITE YOUR NAME CLEARLY USING CAPITAL LETTERS		
FULL NAME	STUDENT ID	DEADLINE July 5, 2022, 13:00
3 QUESTIONS		TOTAL 100 POINTS

By signing below, I pledge that I will write this examination as my own work and without the assistance of others or the usage of unauthorized material or information. I understand that possession of any kind of electronic device during the exam is prohibited. I also understand that not obeying the rules of the examination will result in immediate cancellation and disciplinary procedures.

Signature ....

(20 pts) 1. Consider the tree $(T, <)$ that consists of order embeddings from countable ordinals to $\mathbb{Q}$, that is,

$$T = \{f : \alpha \to \mathbb{Q} \mid \alpha < \omega_1,\ f \text{ is an order-embedding}\}$$

ordered by extension. Show that $height(T) = \omega_1$ and that T is not an Aronszajn tree.

(35 pts) 2. For this question, fix a non-principal ultrafilter $\mathcal{U} \subseteq \mathcal{P}(\omega)$ on ω. A subset $\mathcal{B} \subseteq \mathcal{U}$ is said to generate $\mathcal{U}$ if for every $U \in \mathcal{U}$ there exists $B \in \mathcal{B}$ such that $B \subseteq U$. Consider the cardinal number $d_{\mathcal{U}} = \min\{|\mathcal{B}| : \mathcal{B} \text{ generates } \mathcal{U}\}$.

a) Show that $\aleph_0 < d_{\mathcal{U}} \leq 2^{\aleph_0}$.
b) Show that, under Martin's axiom, we have $d_{\mathcal{U}} = 2^{\aleph_0}$.

(45 pts) 3. Let Fin be the equivalence relation on $\mathcal{P}(\omega)$ defined by

$$A \text{ Fin } B \ \text{ if and only if } \ |A \Delta B| < \aleph_0$$

Let $\mathcal{P}_{\text{inf}}(\omega)$ denote the set of infinite subsets of ω. Consider the forcing poset

$$(\mathbb{P}, \leq, \mathbf{1}) = (\mathcal{P}_{\text{inf}}(\omega)/\text{Fin}, \subseteq^*, [\omega])$$

where $\subseteq^*$ is given by $[A] \subseteq^* [B]$ if and only if $|A \setminus B| < \aleph_0$.

Let $M \models ZFC$ be a countable transitive model. Let $G \subseteq \mathbb{P}^M$ be a $(\mathbb{P}, \leq, \mathbf{1})^M$-generic filter over M. Set $\mathcal{U} = \bigcup G$.

a) Show that $M[G] \models \mathcal{U}$ is a filter on ω.
b) Show that the forcing notion $(\mathbb{P}, \leq, \mathbf{1})$ is ω_1-closed.
c) Show that $M[G] \models \mathcal{U}$ is an ultrafilter on ω.

Fun fact for curious students. $\mathcal{U}$ is indeed a special type of ultrafilter called a *Ramsey* ultrafilter.