M E T U Department of Mathematics

MATH 124 2023-2024 Academic Year Spring Semester Midterm III May 20, 2024, 17:40		
F U L L N A M E	S T U D E N T I D	DURATION 80 MINUTES
3 QUESTIONS ON 2 PAGES		TOTAL 100 POINTS

By signing below, I pledge that I will write this examination as my own work and without the assistance of others or the usage of unauthorized material or information. I understand that possession of any kind of electronic device during the exam is prohibited. I also understand that not obeying the rules of the examination will result in immediate cancellation and disciplinary procedures.

Signature ....

(15+20+15 pts) **1.** In this question, you shall compute the volume of a tetrahedron, i.e. a triangular pyramid, in $\mathbb{R}^3$. Consider the tetrahedron T whose vertices are given by $A(2,1,1)$, $B(1,3,1)$, $C(1,1,4)$ and $D(4,5,6)$. Let P be the plane passing through the points A, B and C. In order for you to imagine these objects easily, below is given a drawing of the tetrahedron T and the plane P.

a) You are given that an equation of the plane P is $6x + 3y + 2z = 17$. Find the point on the plane P that is closest to the point D.

b) Find the area of the triangle $\overset{\triangle}{ABC}$.

c) Recall the volume of a tetrahedron is given by the formula $\frac{1}{3} \times \text{Base area} \times \text{Height}$. Find the volume of the tetrahedron T.

(10+10 pts) **2.** Consider the matrix $M = \begin{pmatrix} 8 & -3 \\ 5 & 0 \end{pmatrix}$.

a) Find the eigenvalues of M. For each of the eigenvalues that you found, find a corresponding eigenvector.

b) Determine whether or not M is diagonalizable. If M is diagonalizable, find an invertible matrix P such that $P^{-1}MP$ is diagonal. If M is not diagonalizable, explain why this is the case.

(10+10 pts) **3.** Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Show that the map $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by $S(x, y) = T((x, y) + T(x, y))$ is a linear transformation.

(10+10 pts) **4.** Find the change of basis matrix from the standard basis $\mathcal{B} = \{(1, 0), (0, 1)\}$ to the basis $\mathcal{E} = \{(3, 5), (1, 1)\}$