

M E T U Department of Mathematics

MATH 124 2023-2024 Academic Year Spring Semester Final Exam June 1, 2024, 09:30		
F U L L N A M E	S T U D E N T I D	DURATION 120 MINUTES
5 QUESTIONS ON 4 PAGES		TOTAL 100 POINTS

By signing below, I pledge that I will write this examination as my own work and without the assistance of others or the usage of unauthorized material or information. I understand that possession of any kind of electronic device during the exam is prohibited. I also understand that not obeying the rules of the examination will result in immediate cancellation and disciplinary procedures.

Signature

(13+12 pts) 1. Consider the lines ℓ_1 and ℓ_2 in \mathbb{R}^3 given by the parametric equations

$$\ell_1 : \begin{aligned} x &= t \\ y &= t, t \in \mathbb{R} \\ z &= t \end{aligned}$$

$$\ell_2 : \begin{aligned} x &= 1 + s \\ y &= -1 - s, s \in \mathbb{R} \\ z &= 1 \end{aligned}$$

a) Show that the lines ℓ_1 and ℓ_2 are skew.

b) Find an equation of a plane P that passes through $(0, 0, 124)$ and that does not intersect ℓ_1 and ℓ_2 .

(15+10 pts) 2. In this question, you shall identify the conic \mathcal{C} given by the equation

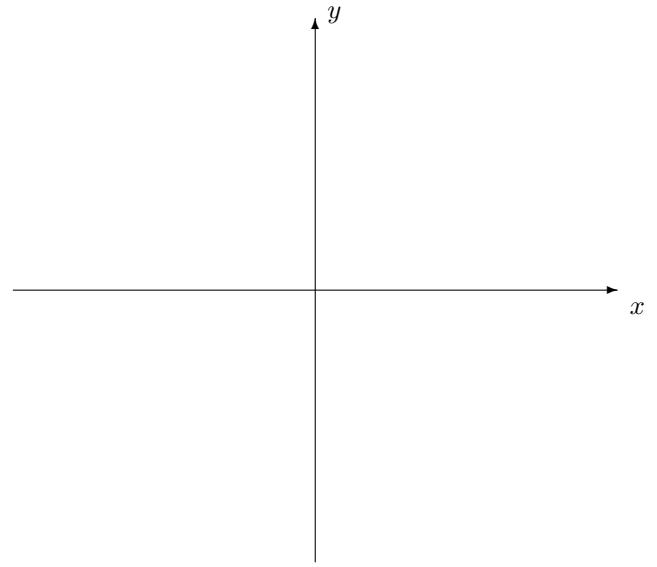
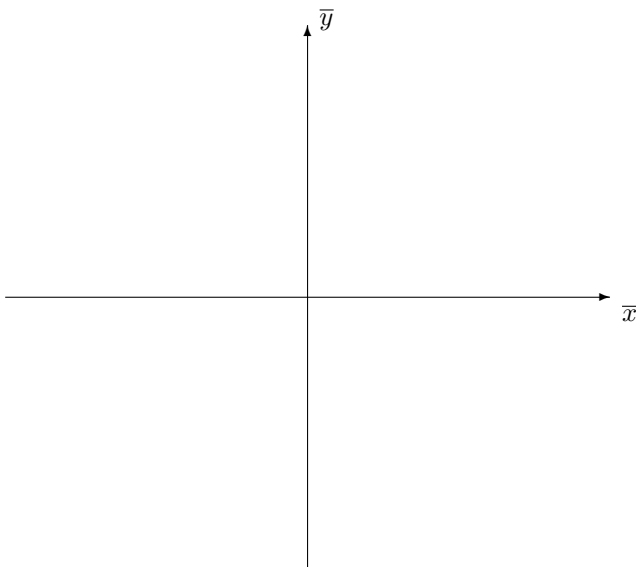
$$9x^2 + 4xy + 6y^2 = 5$$

in cartesian coordinates.

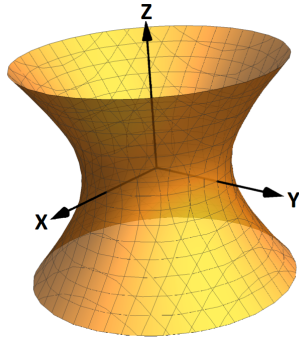
a) Using an appropriate base of change, eliminate the xy -term in this equation, that is, find an $\bar{x}\bar{y}$ -coordinate system so that an equation of \mathcal{C} in this new coordinate system is of the form

$$A\bar{x}^2 + C\bar{y}^2 + D\bar{x} + E\bar{y} + F = 0$$

b) Identify the type of the conic \mathcal{C} and roughly sketch its graph in the $\bar{x}\bar{y}$ -plane and the xy -plane given below.



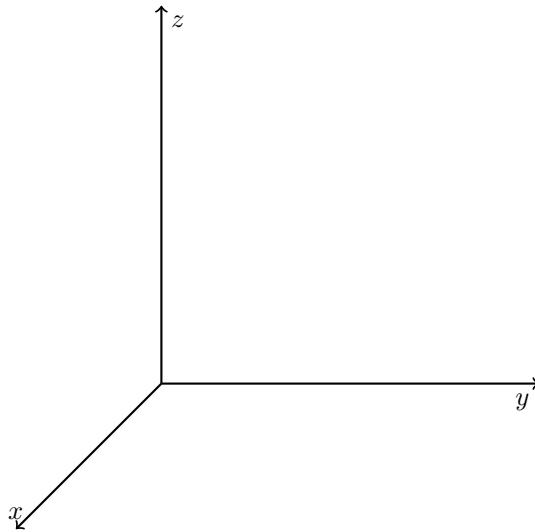
(10+10 pts) 3. The graph of the hyperboloid of one sheet given by the equation $x^2 + y^2 - z^2 = 1$ in cartesian coordinates in the xyz -space is given below.



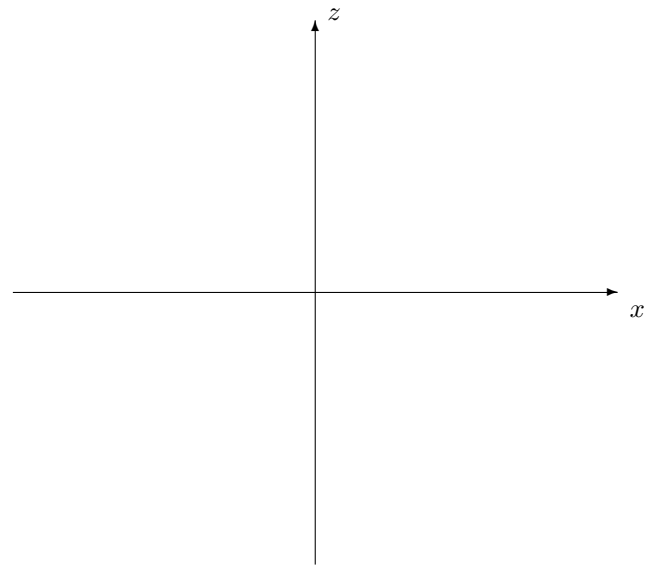
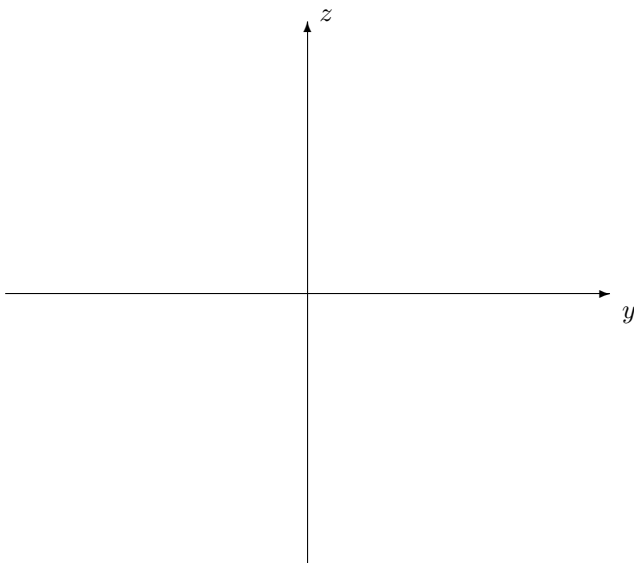
a) Roughly sketch the graph of the hyperboloid of one sheet \mathcal{H} given by the equation

$$x^2 - y^2 + 4y + z^2 - 2z = 4$$

in cartesian coordinates in the xyz -space given below.



b) Sketch the conics that are obtained by intersecting the hyperboloid \mathcal{H} with the planes $x = 0$ and $y = 0$ in the yz -plane and the xz -plane given below respectively.



(15 pts) 4. Consider the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by

$$T(x, y, z) = (x + y + z, 2y - z, x - y + 2z)$$

Recall that we define the kernel of T to be the set $\ker(T) = \{(x, y, z) \in \mathbb{R}^3 : T(x, y, z) = \vec{0} = (0, 0, 0)\}$. Show that $\ker(T)$ is a line and find a parametric equation for this line.

(15 pts) 5. Let \mathcal{C} be the parabola obtained by

- first rotating the parabola with equation $y = x^2$ in the positive direction by $\pi/6$ radians, and
- then translating the resulting parabola in the positive y -direction by 2 units.

Find an equation for \mathcal{C} in cartesian coordinates.