***(13+12 pts)* 1.** Consider the lines ℓ_1 and ℓ_2 in $\mathbb{R}^3$ given by the parametric equations

$$\ell_1 : \quad \begin{aligned} x &= t \\ y &= t, \ t \in \mathbb{R} \\ z &= t \end{aligned} \qquad\qquad \ell_2 : \quad \begin{aligned} x &= 1 + s \\ y &= -1 - s, \ s \in \mathbb{R} \\ z &= 1 \end{aligned}$$

a) Show that the lines ℓ_1 and ℓ_2 are skew.

b) Find an equation of a plane P that passes through $(0, 0, 124)$ and that does not intersect ℓ_1 and ℓ_2.

*(15+10 pts)* **2.** In this question, you shall identify the conic $\mathcal{C}$ given by the equation

$$9x^2 + 4xy + 6y^2 = 5$$

in cartesian coordinates.

a) Using an appropriate base of change, eliminate the xy-term in this equation, that is, find an $\overline{x}\overline{y}$-coordinate system so that an equation of $\mathcal{C}$ in this new coordinate system is of the form

$$A\overline{x}^2 + C\overline{y}^2 + D\overline{x} + E\overline{y} + F = 0$$

b) Identify the type of the conic $\mathcal{C}$ and roughly sketch its graph in the $\overline{x}\overline{y}$-plane and the xy-plane given below.
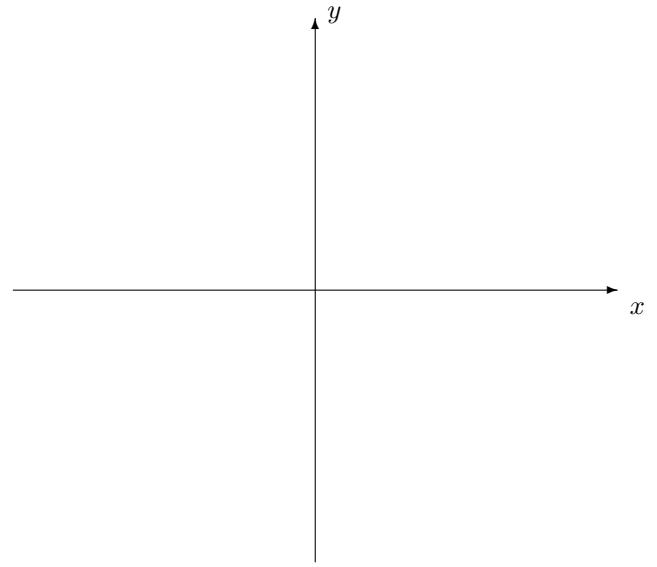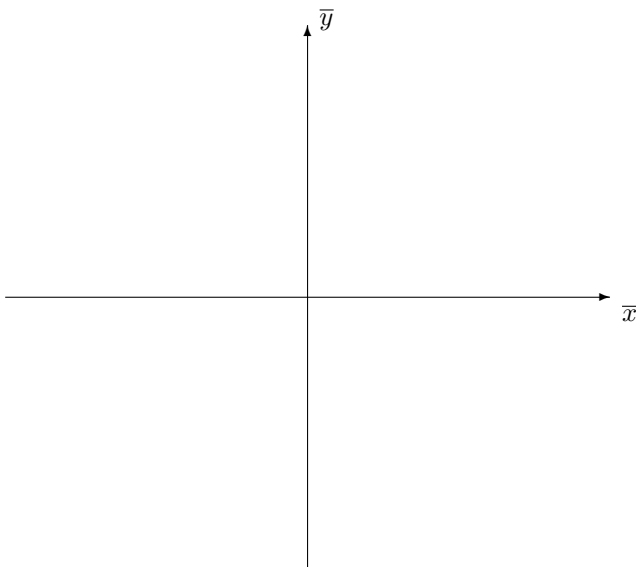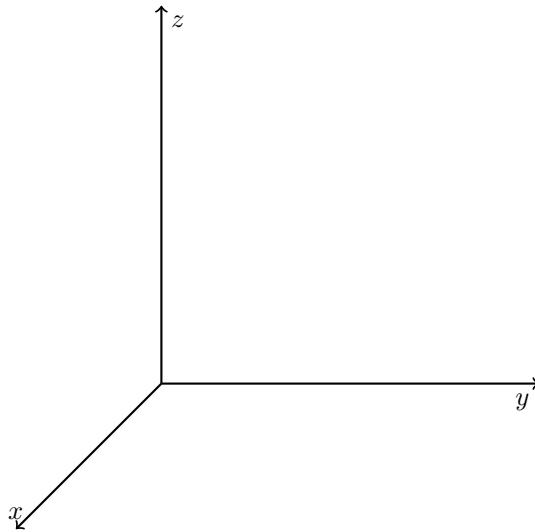
(10+10 pts) 3. The graph of the hyperboloid of one sheet given by the equation $x^2 + y^2 - z^2 = 1$ in cartesian coordinates in the xyz-space is given below.
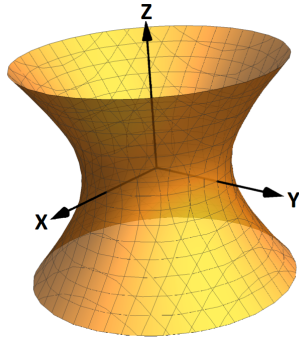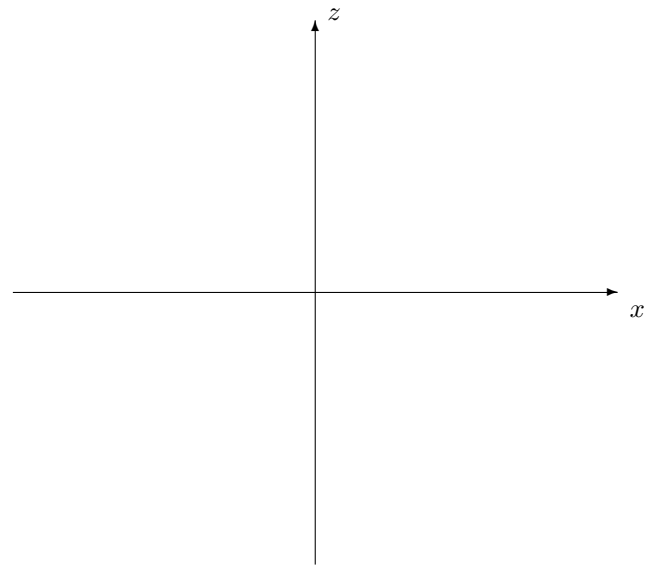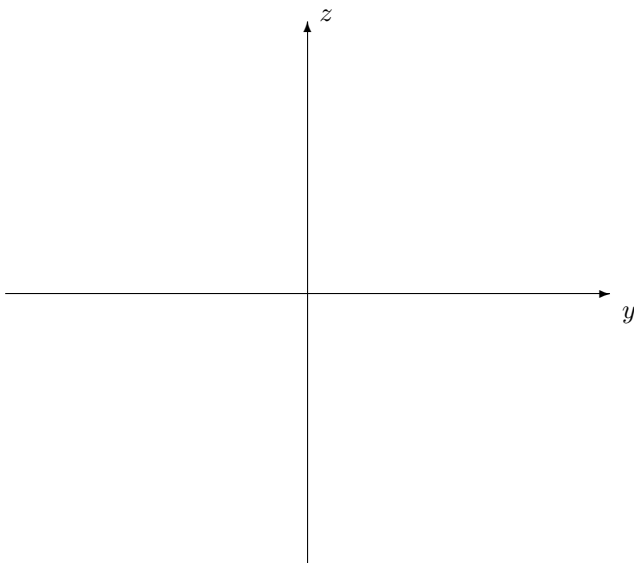

a) Roughly sketch the graph of the hyperboloid of one sheet $\mathcal{H}$ given by the equation

$$x^2 - y^2 + 4y + z^2 - 2z = 4$$

in cartesian coordinates in the xyz-space given below.

b) Sketch the conics that are obtained by intersecting the hyperboloid $\mathcal{H}$ with the planes $x = 0$ and $y = 0$ in the yz-plane and the xz-plane given below respectively.

(15 pts) 4. Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T(x, y, z) = (x + y + z \, , 2y - z, \ x - y + 2z)$$

Recall that we define the kernel of T to be the set $ker(T) = \{(x, y, z) \in \mathbb{R}^3 : T(x, y, z) = \vec{0} = (0, 0, 0)\}$. Show that $ker(T)$ is a line and find a parametric equation for this line.

(15 pts) 5. Let $\mathcal{C}$ be the parabola obtained by

- first rotating the parabola with equation $y = x^2$ in the positive direction by $\pi/6$ radians, and

- then translating the resulting parabola in the positive y-direction by 2 units.

Find an equation for $\mathcal{C}$ in cartesian coordinates.