M E T U Department of Mathematics

Math 123, Fall 2023, Final Exam, January 13, 2024, 13:30		
FULL NAME	ID NUMBER	SIGNATURE
7 QUESTIONS ON 4 PAGES		DURATION: 120 MINUTES

Q1.(20 points) If they exist, find all **positive** integer solutions of the linear Diophantine equation

$$13x + 5y = 250$$

Q2.(13 points) Let a, b be integers such that $\gcd(a, b) = 1$. Show $\gcd(4a+3b, a+b) = 1$.

Q3.(12 points) Find the greatest integer k such that $24^k \mid 123!$.

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Q4.(15 points) a) Complete the following statement of the Möbius inversion formula: Let F and f be two number-theoretic functions related by the formula $F(n) = \sum_{d|n} f(d)$. Then ...

b) You are given that there exists a unique number-theoretic function f satisfying

$$n^2 = \sum_{d|n} f(d)$$

for all positive integers n. Find $f\left(7^{123}\right)$.

Q5.(10 points) Using Euler's theorem, find the integer $0 \leq r < 1400$ that satisfies $11^{962} \equiv r \pmod{1400}$.

Q6.(14 points) Let a, b be positive integers. Show that if $a \mid b$, then $\phi(a) \mid \phi(b)$.

Q7.(16 points) Alice has the public RSA key $(n, k) = (1147, 7)$. You are given the fact that $1147 = 31 \cdot 37$.

a) Encrypt the plaintext $M = 10$ using the RSA algorithm with Alice's public key.

b) Find Alice's private key. Determine the recovery exponent j and explain how Alice would decrypt the ciphertext obtained in Part (a).