MATH 535, Topology, Homework 5

1. (3 (if) +2 (and only if) pts)

Let X be a topological space. Show that X is a T_6 if and only if X is T_4 and every closed subset of X is a G_δ-set.

Warning. Recall that the definition of a T_6-space may differ from source to source. Our definition is that X is T_6 if X is T_1 and for every disjoint closed subsets $A, B \subseteq X$ there exists a continuous function $f : X \to [0, 1]$ with $A = f^{-1}(0)$ and $B = f^{-1}(1)$. Some books use the other part of this iff statement as the definition. The point of this question is to prove the equivalence of these definitions.

2. (2 pts) Show that every metrizable Lindelöf space is second-countable.

3. (3 pts) A topological space X is said to be a locally metrizable space if every point has an open neighborhood whose subspace topology is metrizable. Show that if X is a locally metrizable compact Hausdorff space, then X is metrizable.

Solutions