MATH 535, Topology, Homework 1

1. (2+3+3 pts)

Let \overline{G} be a group. Our aim in this question is to first endow $\mathcal{P}(G)$ with a topology and then construct the space of subgroups of G as a subspace of $\mathcal{P}(G)$.

For each **finite** subset $F, K \subseteq G$, let $U_{F,K} = \{S \in \mathcal{P}(G) : F \subseteq S, K \subseteq S^c\}$.

a) Show that the collection

 $\mathcal{B} = \{ U_{F,K} \subseteq \mathcal{P}(G) : F, K \text{ are finite subsets of } G \}$

is a basis for a topology on $\mathcal{P}(G)$.

For the remainder of this question, you will consider $\mathcal{P}(G)$ with the topology $\tau_{\mathcal{B}}$ where $\tau_{\mathcal{B}}$ is the topology generated by the basis \mathcal{B} in part a.

b) Show that the set

 $\operatorname{Sub}(G) = \{ H \in \mathcal{P}(G) : H \text{ is a subgroup of } G \}$

is a closed subset of $\mathcal{P}(G)$.

Hint. Show that the complement of this set is open. In order to do that, ask yourself this: When is a subset $H \subseteq G$ not a subgroup? Is the set of H's which do not contain the identity an open set? Is the set of H's which do not contain the inverse of some element an open set? Is the set of H's which in not closed under group multiplication an open set?

For the last part of this question, suppose that $G = \mathbb{Z}$ and consider $Sub(\mathbb{Z})$ with the subspace topology as a subspace of $\mathcal{P}(\mathbb{Z})$.

c) Show that $\lim_{n\to\infty} n\mathbb{Z} = \{0\}$ in the topological space $\operatorname{Sub}(\mathbb{Z})$.

2. (2 pts) Let X be a topological space and $A \subseteq X$. Prove that if A is closed, then A' is closed.

3. (2 pts) Let (X, \preceq) be a linearly ordered set. Prove that the corresponding order topology on X is Hausdorff.

Solutions