SEES 503
SUSTAINABLE WATER RESOURCE
FLOOD ROUTING

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Overview

- Introduction
- Storage Equation
- Reservoir Routing
- Routing in Natural Channels
Flood routing is a procedure through which the temporal variation of discharge at a point on a stream channel or on a reservoir may be determined by consideration of similar data from a point upstream.

Change in flood hydrograph due to storage

Flood routing is a process, which shows how a flood wave is reduced in magnitude, and lengthened in time by the storage in a reach or reservoir between the two points.
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Storage (Continuity) Equation

\[ \Delta S = \sum (I - Q) \Delta t \]

or

\[ I - Q = \frac{dS}{dt} \]

\( I \) : inflow

\( Q \) : outflow

\( \frac{dS}{dt} \) : change in storage

\[ \bar{I} - \bar{Q} = \frac{\Delta S}{\Delta t} = \frac{S_2 - S_1}{\Delta t} \]

\( \bar{I} \) : the average inflow \( (I_1 + I_2)/2 \) during \( \Delta t \).

\( \bar{Q} \) : the average outflow \( (Q_1 + Q_2)/2 \) during \( \Delta t \).

\( S_1 \) and \( S_2 \) : storage at the beginning and at the end of \( \Delta t \).
Although the storage equation is correct and simple, the movement of a flood wave in a river channel or in a reservoir is a condition of unsteady flow.

Therefore following **assumptions** and **approximations** are required.

- The stream should be divided into parts (reaches) where channel characteristics are constant or assumed to be constant.
- There should be a stream gauge at each end of the reach.
- The time interval, $\Delta t$, should be as large as possible so as to have less number of computation steps, but at the same time it should be small enough to observe the passage of the flood peak.
Assumptions and approximations are required. (con’t)

- If there are tributaries entering the reach whose discharges are small compared to the inflow, they may be ignored.

\[ q_1 : \text{add to inflow before routing} \]
\[ q_3 : \text{add to outflow after routing} \]

\[
\begin{align*}
q_2 : & \quad \frac{L_u}{L_u + L_d} q_2 \quad \text{add to inflow before routing} \\
q_2 : & \quad \frac{L_d}{L_u + L_d} q_2 \quad \text{add to outflow after routing}
\end{align*}
\]
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Reservoir Routing

- Reservoir routing provides methods for evaluating the effects of a reservoir on a flood wave passing through it.
- For the design and planning of hydraulic structures, it applies to the determination of the location and the capacity of reservoir, of the size of outlets or spillways, etc.

In reservoir routing, it is assumed that the water surface in the reservoir is horizontal at all times and the relationship between the storage and discharge is constant.
Reservoir Routing

\[(I_1 + I_2) + \left(\frac{2S_1}{\Delta t} - Q_1\right) = \frac{2S_2}{\Delta t} + Q_2\]

Known

Unknown

Known

Obtained
Reservoir Routing

\[
(I_1 + I_2) + \left( \frac{2S_1}{\Delta t} - Q_1 \right) = \frac{2S_2}{\Delta t} + Q_2
\]

If not known \( Q_0 = I_0 \)

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<tbody>
<tr>
<td>Time</td>
<td>Inflow</td>
<td>( I_0 + I_1 )</td>
<td>( \frac{2S_1}{\Delta t} - Q_1 )</td>
<td>( \frac{2S_2}{\Delta t} + Q_2 )</td>
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## Reservoir Routing

If \( Q_n > I_n \): to complete the outflow hydrograph after the end of the inflow hydrograph, the values of inflow are taken to be constant and equal to the last inflow value \( I_n \) and routing process is continued till an outflow value equal to or closer to \( I_n \) value is obtained. This way the lengthening on the base time can be determined.

\[
(I_1 + I_2) + \left( \frac{2S_1}{\Delta t} - Q_1 \right) = \frac{2S_2}{\Delta t} + Q_2
\]

If not known \( Q_0 = I_0 \)

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Routing in Natural Channels

In natural channels, storage is not only a function of outflow but also inflow.

Wedge storage may be positive or negative depending upon whether the flood in coming toward the reach (+) or going away from the reach (-).

Storage in natural channels

Reach 1: Rising stage (flood wave is coming to)
prism storage = ABCE
wedge storage = AED

Reach 2: Falling stage (flood wave is going away)
prism storage = A'B'C'E'
wedge storage = A'E'D'
Routing in Natural Channels

- (gain to storage during rising) > (loss from storage during falling)
- The routing procedure in the channels is obtained by expressing storage as a function of both outflow and inflow.

Change in flood hydrograph due to storage in the channel

Outflow vs storage
Routing in Natural Channels

- **Muskingum Method** is generally used in the channel routing procedure.

\[ S = KQ + Kx(I - Q) \]
\[ S = K(xI + (1 - x)Q) \]

\( K \): travel time of the center of mass of flood wave from the upstream end of the reach to the downstream end.

\( x \): dimensionless constant. It indicates the effective weights of inflow and outflow.

**TRIAL & ERROR METHOD**
Assume \( x \) plot \( S \) vs \([xI+(1-x)Q]\)
The correct \( x \) is the one that gives closest to the straight line.
\( K \): slope of this line.

Determination of \( x \) by trial and error
Routing in Natural Channels

- **Muskingum Method**

\[ S_1 = K \left( xI_1 + (1-x)Q_1 \right) \]

\[ S_2 = K \left( xI_2 + (1-x)Q_2 \right) \]

\[ (I_1 + I_2) + \left( \frac{2S_1}{\Delta t} - Q_1 \right) = \frac{2S_2}{\Delta t} + Q_2 \]

\[ (I_1 + I_2) + \left( \frac{2K(xI_1 + (1-x)Q_1)}{\Delta t} - Q_1 \right) = \frac{2K(xI_2 + (1-x)Q_2)}{\Delta t} + Q_2 \]
Routing in Natural Channels

- **Muskingum Method**

\[
(I_1 + I_2) + \left( \frac{2K(xI_1 + (1-x)Q_1)}{\Delta t} - Q_1 \right) = \frac{2K(xI_2 + (1-x)Q_2)}{\Delta t} + Q_2
\]

Rearranging above equation for inflow and outflow terms

\[
\frac{2K(1-x) + \Delta t}{\Delta t} Q_2 = \frac{\Delta t - 2Kx}{\Delta t} I_2 + \frac{\Delta t + 2Kx}{\Delta t} I_1 + \frac{2K(1-x) - \Delta t}{\Delta t} Q_1
\]

\[
Q_2 = \frac{\Delta t - 2Kx}{2K(1-x) + \Delta t} I_2 + \frac{\Delta t + 2Kx}{2K(1-x) + \Delta t} I_1 + \frac{2K(1-x) - \Delta t}{2K(1-x) + \Delta t} Q_1
\]

\[
Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1
\]

\[
C_0 + C_1 + C_2 = 1
\]
Routing in Natural Channels

- **Muskingum Method**

## Channel Routing

<table>
<thead>
<tr>
<th>(1) Time (hr)</th>
<th>(2) I (m³/s)</th>
<th>(3) C₀I₂</th>
<th>(4) C₁I₁</th>
<th>(5) C₂Q₁</th>
<th>(6) Q (m³/s)</th>
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<tr>
<td>t₀</td>
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